DERIVATIONS OF DIFFERENTIALLY PRIME RINGS

IMAN AHMAD TAHA

UNIVERSITI PENDIDIKAN SULTAN IDRIS

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ABSTRACT

The concept of derivation on rings is an old problem and plays a very important role in algebra, analysis, geometry and physics. The concept of derivation was generalized in different directions as centralizing derivation, Lie and Jordan derivations, (θ, φ)-derivation, generalized derivations and reverse derivations. The main aim of this research is to generalize these concepts. Therefore, the action of (θ, φ)- derivations on Lie and Jordan Ideals of prime rings with characteristic not equal two will be investigated. The next objective was to determine the commutativity of a prime ring when the generalized derivation acting on the Jordan ideal of a prime ring. Furthermore, this research will extend the centralizing derivations and reverse derivation on δ- ideal for δ- prime rings. Finally, we extended the concepts of generalized reverse derivation and generalized reverse (θ, φ)- derivation on a δ- prime and δ- semiprime rings. Some techniques and methods of the theory of commutative rings will be significantly used in this research, such as the properties of commutators and centralizing derivations. The findings of the research showed that the prime ring, R is commutative if there is a generalized derivation with a specific condition on R. Also, some results depending on a generalized (θ, φ)- derivation on Jordan ideal of R are extended. Beside, the relationship between some kinds of derivations as centralizing δ- derivations and the commutativity of δ- prime rings was developed. Finally, the commutativity of δ- prime ring with centralizing reverse δ- derivation was obtained. It can be concluded that the study of derivation and reverse derivation on a prime and δ- prime rings have considered some conditions to obtain the proven results. As an implication, the obtained results in this research can be used as a guide for other researchers to extend new results related to semiprime rings in the future.

TERBITAN BAGI GELANGGANG PERDANA TERBEZA

ABSTRAK

Konsep terbitan ke atas gelanggang merupakan suatu permasalahan yang telah ada sejak dahulu lagi dan memainkan peranan yang penting dalam aljabar, analisis, geometri dan fizik. Konsep terbitan telah digeneralisasikan dalam pelbagai arah sebagai terbitan pemusatan, terbitan Lie dan Jordan, (θ, φ)- terbitan, terbitan tergeneralisasi dan terbitan terbalik. Matlamat utama penyelidikan ini adalah untuk mendapatkan generalisasi bagi konsep-konsep ini. Oleh itu, tindakan bagi (θ, φ)-terbitan ke atas ideal Lie dan ideal Jordan bagi gelanggang perdana dengan cirian tidak sama dengan dua akan dikenal pasti. Objektif berikutnya adalah untuk menentukan tukar tertib bagi gelanggang perdana apabila terbitan tergeneralisasi bertindak ke atas ideal Jordan bagi gelanggang perdana. Seterusnya, penyelidikan ini akan memperluaskan terbitan pemusatan dan terbitan terbalik ke atas δ- ideal bagi δ- gelanggang perdana. Akhir sekali, kami telah memperluaskan konsep generalisasi terbitan terbalik dan generalisasi (θ, φ)- terbitan terbalik ke atas gelanggang δ-perdana and δ- semiperdana. Beberapa teknik dan kaedah bagi teori gelanggang tukar tertib seperti, sifat penukar tertib, terbitan pemusatan akan digunakan secara signifikan di dalam penyelidikan ini. Dapatan kajian ini menunjukkan bahawa gelanggang perdana, R adalah tukar tertib jika wujud terbitan tergeneralisasi dengan syarat khusus ke atas R. Selain itu, beberapa hasil dapatan yang bergantung ke atas (θ, φ)- terbitan tergeneralisasi ke atas ideal Jordan bagi R dapat diperluaskan. Di samping itu, hubungan antara beberapa jenis terbitan sebagai pemusatan terbitan δ dan tukar tertib bagi δ- gelanggang perdana telah dibangunkan. Akhirnya, tukar tertib bagi δ- gelanggang perdana dengan memusatkan δ- terbitan terbalik telah diperoleh. Boleh disimpulkan bahawa kajian terbitan dan terbitan terbalik pada gelanggang perdana dan δ- gelanggang perdana telah mempertimbangkan beberapa syarat untuk mendapatkan hasil yang telah dibuktikan. Sebagai implikasi, hasil yang diperoleh dalam penyelidikan ini boleh dijadikan panduan kepada pengkaji lain untuk memperluaskan hasil lain yang berkaitan dengan gelanggang semiperdana pada masa akan datang.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF SYMBOLS

R	-	Ring
$Z(R)$	-	Center of the ring R
$C_R(S)$	-	Centralizer of a subset S in a ring R
$\mathbb{Z}_n$	-	Ring of integers mod n
δ	-	Derivation
∂_a	-	Partial Derivation of a
I_a	-	Inner Derivation of a
F	-	Generalized Derivation
Der R	-	The set of all derivations
Δ	-	The subset of Der R
$[u, v]$	-	Lie Commutator of u and v
$C(R)$	-	Set of all commutators in R
$(u \circ v)$	-	Jordan Commutator of u and v
$\in$	-	Element of
$charR \neq 2$	-	characterastic of R non equal 2
$A \subset R$	-	A is a proper subset of R
$A \subseteq R$	-	A is a subset of R or equal to

$A \nsubseteq R$	-	A is not contained in R neither equal to
$\notin$	-	Not element of
$\neq$	-	Not equal to

CHAPTER 1

INTRODUCTION

1.1 Introduction

The theory of ring is a masterpiece of mathematical unification, putting together many branches to create a strong machine for solving historical problems in mathematical sciences. A ring is an abelian group under addition, closed under multiplication, and the two distributive laws are hold as well. A ring R is named an associative ring if the second operation (multiplication) defined on R is associative. A subring is a subset of a ring which is closed under subtraction and multiplication.

Ring elements may be numbers such as real, rational numbers, but they may also be non numerical objects, such as square matrices and functions. An ideal of a ring is a special subset of its elements, so we can say A is an ideal in R if A is a subring of R and $Ar \subseteq A$ or $rA \subseteq A$, for all $r \in R$ (Dummit & Foote, 1991). An ideal A of R is nilpotent if $A^n = \{0\}$, for some $n \in \mathbb{N}$ (Grillet, 2007). Let R be a ring. The additive subgroups A, B of R are called

(i) A Lie ideal of R, if $[a, r] \in A$, where $[a, r] = ar - ra$

(ii) A Jordan ideal of R, if $(b \circ r) \in B$, where $(b \circ r) = br + rb$,

$\forall a \in A, b \in B$ and $r \in R$ respectively. (Jordan & Jordan, 1978)

Now, we say R is a prime ring if (Dummit & Foote, 1991)

(i) L and M are two ideals on R, $LM = \{0\}$, then $L = \{0\}$ or $M = \{0\}$.

(ii) L and M are two right ideals on R, $LM = \{0\}$, then $L = \{0\}$ or $M = \{0\}$.

(iii) L and M are two left ideals on R, $LM = \{0\}$, then $L = \{0\}$ or $M = \{0\}$.

Using these conditions, it can be checked that for any integral domain, a matrix ring over an integral domain is a prime ring. In particular, the ring of integer matrices is a prime ring. The ring R is called a semiprime if the zero ideal is a semiprime ideal (Lam, 2012).

There is a relationship between differentiation rings and their endomorphisms and automorphisms, which constitute a group of endomorphisms and automorphisms of this ring. Hence the structure of the different classes of associative rings, their relationship with the various classes of rings and Lie algebras can be determined.

The concept of the derivation on a ring R is a mapping $\delta : R \mapsto R$, where $\delta(a+b) = \delta(a)+\delta(b)$ and $\delta(ab) = \delta(a)b + a\delta(b),\ \forall a, b \in R$ (Posner, 1957). The set of all derivations of a ring R is denoted by $\operatorname{Der} R$. If $u \in R$ is a fixed element, then a map $\partial_u : R \to R$ given by $\partial_u(r) = [u, r] = ur - ru,\ \forall r \in R$ or $I_u(r) = [u, r]$ is a derivation of R. (In this case, ∂_u or I_u are called inner derivation of R generated by u). A ring R is named a differentially ring if it is equipped with a derivation, this means, a ring is differentially when a derivation δ is defined on R. So, once we defined a derivation on a ring, then this ring is said to be a differentially ring (Herstein, 1978).

Let Δ be a subset of $\operatorname{Der} R$. An ideal C on R, which $\delta(C) \subseteq C$ for $\delta \in \Delta$ is called a δ-ideal. In addition, a ring R is a δ-prime if, for any δ-ideals C, D of R, the condition $CD = \{0\}$ implies that $C = \{0\}$ or $D = \{0\}$ (Artemovych & Lukashenko, 2015). This research will focus on the derivations, generalized derivation, (θ, φ)-derivation, centralizing derivation and reverse derivation in specific rings such as a prime rings and δ-prime. Furthermore, in this research, the commutator and derivation properties and many facts about δ- ideal, δ-semiprime rings and δ-prime rings will be used as a method extensively.

Let δ be a derivation on R and θ, φ are maps on R, a map δ is named (θ, φ)-derivation if $\delta(a+b) = \delta(a) + \delta(b)$ and $\delta(ab) = \delta(a)\theta(b) + \varphi(a)\delta(b),\ \forall a, b \in R$ and δ be a left (θ, φ)- derivation if $\delta(ab) = \theta(a)\delta(b) + \varphi(b)\delta(a),\ \forall a, b \in R$ (Ashraf, 2005). In this research the action of (θ, φ)- derivation acts on the ideal and Lie ideal of prime rings, will be investigated and also to apply this derivation for Jordan ideal, which is a subring of a prime ring.

After that, the generalized derivation G will be extended for Lie and Jordan ideals of prime rings, which G is a map on R, where

$$G(n+m) = G(n) + G(m)$$

$$G(nm) = G(n)m + n\delta(m), \qquad \forall n, m \in R$$

such that δ is a derivation on R (Hvala, 1998).

Finally, another concepts have been highlighted through the last chapter of finding results. These concepts are related to the generalized reverse derivation and generalized reverse (θ, φ)- derivation on a δ- prime and δ- semiprime rings. In the light of that, we can say, the additive mapping G is said to ba a generalized reverse derivation related to a non zero reverse derivation on a ring R when the condition $G(ab) = G(b)a + b\delta(a)$, for any a, b in R is fulfilled. Also, the additive map δ on a prime ring is called a reverse (θ, φ)- derivation when the condition $\delta(ab) = \delta(b)\theta(a) + \varphi(b)\delta(a)$ is met for any two elements a, b in the ring R. Furthermore, the

additive map G on a prime ring R is named a generalized reverse (θ, φ)- derivation, related to a reverse (θ, φ)- derivation δ, if the condition $G(ab) = G(b)\theta(a) + \varphi(b)\delta(a)$ is satisfied for any two elements a, b in the ring R (Ibraheem, 2016).

1.2 Research Background

During this paragraph, we will show the historical sources and the researches that have a directly related to our topic, which we posted that as the background of this work. Fraenkel (1915), Noether (1921) in addition to Hilbert (1932) are the ones who contributed in expansion concepts on rings. For more details, the first primary definition of a ring was given by Fraenkel (1915). Noether (1921) gave a new definition of commutative rings and improved the foundations of commutative ring theory. Most of all research on algebra came up around 1940 by Jacobson (1943). Many researchers started studying prime rings and expanded this concept by applying mappings on them. Posner (1957) collected between the derivations and the prime rings, Jordan derivation has been applied to a prime ring in Herstein (1957), Awtar (1973) studied Lie and Jordan ideals in prime rings as well, and Chuang (1989) focused on nilpotent derivations on prime rings. There were authors completed their research about rings, such as Amitsur (1971), Rowen (1974) and France-Jackson (1993). A lot of researchers have applied the derivation concepts on prime and semiprime rings, such as Yenigul and Argac (1994), who combined the concept of derivation with prime

and semiprime notions, and other researchers focused on semiprime rings with the reverse derivation, such as Vukman (1996), Samman and Thaheem (2003), as well as Samman and Alyamani (2007).

The study of the differentiation concept is one of the most worthy and mysterious ideas in mathematics. It came to mathematics for a long time, to be recently what was achieved in analysis, from this point, the borders of applications of this concept have been extending continuously.

Overall, the derivations on rings are not undergoing wide research yet. Dedekind was the first researcher who applied the concept of differentiation in algebra in the 19^{th} century, and these ideas can be seen in Landau (1903), but it was found later that Jacobson (1937) introduced the definition of the derivation more clearly. Ritt (1950), who started with well-known books on differential algebra, even though, he was an analysis researcher, but his passion was towards algebra subjects. So, the Ritt algebra, which we can find it in differentially ring, is a notion has carried his name upon his hardworking in this field of study. Kaplansky (1956) and Kolchin (1973) are some researchers in this field, which their work were the fundamental stone in this domain. They made the subject easier and accessible for mathematician society. After that, Posner (1957) who had come to study about derivation of associative rings, which is a development for differentiation. This concept has been gotten an extremely role after Posner's theorems. He proved that if R is a prime ring with $charR \neq 2$ and there exists two derivations, then one of these have to equal zero. Then, he proved the

commutativity of a prime ring for a centralizing derivation. Herstein (1957) initiated the study of Jordan derivation where he introduced the reverse derivation concept with considering a Jordan derivation on a prime ring R. Furthermore, he proved the commutativity of a ring R when there is a non zero reverse derivation δ on this ring, while Martindale (1964) studied about Lie derivation in the same field. The question has aroused, if a mapping is acting as a derivation on Lie subring of a prime ring, it is come from an ordinary derivation to be an important problem posted by Herstein (1970). Then the researchers who studied in this field are Vukman (1996) and Ozge *et al.* (2018).

Nowicki (1985) has studied the Lie ring of derivations of D-simple commutative rings. The studies of derivations on rings can be found in previous works, such as in Ashraf (2005) and Brešar *et al.* (2007). Aboubakr and Gonzalez (2015) extended the reverse derivation on a ring and more than ever, Ibraheem (2016) focused on the right ideal U on a prime ring R, which the common elements between any ideal and the center of R is not trivial set, and if G is a generalized reverse derivation, related to a reverse derivation δ on R, then, the commutativity of R when $[G(a), a] \in Z(G)$, for any $a \in U$, are fulfilled. Moreover, Huang (2019) expanded special condition for a generalized reverse derivation on a prime ring to be commutative. Finally, the studies of derivations on rings can be found in previous works, such as in Ashraf (2005), Martindale (1964), Vukman (1996), Brešar *et al.* (2007) and Al khalaf *et al.* (2019). However, the studies on derivation are still few so far. Therefore, this research will extend some theorems of derivation and reverse

derivation concepts on prime rings and δ- prime rings to obtain new results.

1.3 Problem Statements

The properties of the derivations can be, on the one hand, a deep study of the structure of associative ring. On the other hand, by allowing differentiation, some new interesting and important properties of prime and semiprime rings can be determined, and on this basis can deepen our knowledge of their structures.

Many researchers have extensively studied derivations on rings, and some of these results have been extended, for example, Yenigul and Argac (1994) extended result developed by Bell and Kappe (1989) for γ- derivation in prime rings. Daif and Bell (1992) studied the derivation δ for a non zero ideal A on a semiprime ring R, which $\delta([n,m]) = [n,m], \forall n, m \in A$, then $A \subseteq Z(R)$. Ashraf (2005) generalized the result for left (θ, φ)- derivation in prime rings R. Quadri *et al.* (2003) extended the result to a generalized derivation. Zaidi *et al.* (2004) applied some conditions on a derivation, which is acting on a Jordan ideal such that the ring is commutative. In this direction, this research will extend Zaidi's result for the action of a (θ, φ)- derivation on Lie and Jordan ideals. Also, in this research, the above mentioned results will be extended to a more general situation by using Golbasi and Aydin (2002) results, this means, when the left (θ, φ)- derivation is applied on Lie and Jordan ideals.

There are various results related to the commutativity of a ring, and the presence of particular derivations on a prime ring R. Awtar (1973) proved that if R is a prime ring with $charR \neq 2$ have a derivation δ and a Jordan ideal V, where the derivation is centralizing on V, that implies $V \subseteq Z(R)$. Moreover, Mayne (1976) obtained the same result for centralizing automorphisms. Also, Mayne (1982) has shown that if the centralizing derivation or automorphism acts for an ideal on a prime ring, then the ring is commutative. Trzepizur (1984) proved a similar result for a semiprime ring. Now, the question is how does the centralizing derivation act on non-zero δ-ideal of a δ-prime ring. The studies of derivations on rings can be found in previous works, such as in Ashraf (2005), Martindale (1964), Vukman (1996), Brešar *et al.* (2007) and Al khalaf *et al.* (2019). Secondly, what is the relationship between the generalized derivation and a presence of the commutativity of a prime ring, then some conditions will be applied on the generalized derivation G associated with a non zero (θ, φ)- derivation δ on a prime ring. For the next chapter, the results when a centralizing and reverse derivations are applied on a δ- prime rings for a δ- ideal in R will be presented.

Lastly, Bresar (1991) extended the derivation to a generalized derivation. Also, Ozge *et al.* (2018) and Huang (2019) studied the ring with a generalized reverse derivation, related to a non zero reverse derivation, which some conditions on a non zero ideal are satisfied to obtain the commutativity of this ring. In addition, other researchers extended a few properties about the commutativity of a prime ring and semiprime ring as well like Rehman (2002). In this thesis, we worked to find

out whether the prime and semiprime rings will keep on their commutativity with a generalized reverse derivation or a generalized reverse (θ, φ)- derivation.

The motivations of the research that lead to all works on this thesis are listed in Table 1.1.

Table 1.1

The Motivation of the Research

Chapters	**Motivation of the Research**
4. Derivation on Lie and Jordan ideals of a prime ring	Extend Theorem 2.19 and 2.20 with Lie ideal instead of ideal or right ideal via Theorem 4.2. Also, to extend Theorem 2.17 with Jordan ideal using (θ, φ)-derivation instead left (θ, θ)-derivation of a ring R by Theorem 4.3, in addition to extend it with left (θ, φ)-derivation by Theorem 4.4, then using (θ, θ)-derivation instead left (θ, θ)- derivation on R through Theorem 4.5
5. The commutativity of prime ring for generalized derivation on prime ring	To put a specific condition on a generalized derivation associated with a non zero derivation on a ring to obtain the commutativity of this ring via Theorem 5.1. Also, to extend Theorem 4.5 with Jordan ideal using generalized (θ, θ)-

(continue)

Table 1.1 *(continue)*

Chapters	**Motivation of the Research**
	derivation on a ring R via Theorem 5.2. In addition to extend Theorem 2.17 through Theorem 5.3, Theorem 5.4 and Theorem 5.5.
6. The centralizing derivation on a δ- ideal of a δ-prime ring	To generalize Theorems 2.2 and 2.15 by putting a specific condition for centralizing derivation on δ- ideal of a δ- prime ring to conclude the commutativity of this ring through Theorem 6.1.
7. The centralizing reverse derivation on a δ- ideal of δ- prime rings	To extend Theorem 6.1 by applying the centralizing reverse derivation on δ- ideal of a δ- prime ring via Theorem 7.2.
8. Generalized reverse derivation in δ- prime rings and δ- semiprime rings	To generalize Theorems 7.1 and 7.2 by changing the taken condition that related to a generalized reverse derivation and generalized reverse (θ, φ)- derivation on a prime and semiprime ring, all of that via the theorems 8.1, 8.2 and 8.3 and 8.4 to extend these four theorems into the theorems 8.5, 8.6 and 8.7.

1.4 Research Objectives

The objectives of this research are:

(i) To investigate the action of (θ, φ)- derivations for Lie and Jordan Ideals on the prime ring.

(ii) To examine the commutativity of a prime ring when the generalized derivation acts on the Jordan ideal.

(iii) To extend the centralizing derivation for δ- ideal for δ- prime ring.

(iv) To extend the centralizing reverse derivation for δ- ideal in δ- prime ring.

(v) To develop the generalized reverse and generalized reverse (θ, φ)- derivations, related to a reverse derivation and reverse (θ, φ)- derivation on a δ- prime and δ- semiprime rings.

1.5 Research Questions

The main questions of this research are:

(i) What are the results of the action of (θ, φ)- derivations for Lie and Jordan Ideals on prime rings?

(ii) Is the prime ring R will keep its commutativity when the generalized derivation acting on the Jordan ideal of a prime ring?

(iii) How does the centralizing derivation act on non zero δ- ideal of δ- prime ring?

(iv) How does the centralizing reverse derivation act on non zero δ- ideal of δ- prime ring?

(v) Is the δ- prime ring or δ- semiprime ring R will keep its commutativity when the generalized reverse derivation and generalized reverse (θ, φ)- derivation are applying on the ring?

1.6 Scope of the Study

This research concentrates on studying the derivation and reverse derivation on prime, δ- prime and δ- semiprime rings, and how a (θ, φ)- derivation is acting on the Lie and Jordan ideals of prime and δ-prime rings. In addition, the focus of this research will be on centralizing reverse derivation for δ- ideal on a δ- prime ring. Also, the working will focus on generalized derivation and generalized reverse derivation, related to a reverse derivation. Furthermore, the generalized reverse (θ, φ)- derivation, affined to a reverse (θ, φ)- derivation will be discussed.

1.7 Significance of Findings

Lie theory finds a great role in the study of symmetry of this world (in modern physics, chemistry, manufacturing, and so on), which has different practical steps in finding solutions to various differential equations. In addition, it can describe the various phenomena in the construction, geology, geodesy, oil industry and so forth.

Many results have been found by other researchers in this field. For example, in 1940, it was found that the Galois Theory of algebraic equations may be transferred to the theory of ordinary linear differential equations.

Overall, the importance of this research is to study the derivation, reverse derivation, generalized derivation, (θ, φ)- derivation, centralizing reverse derivation, generalized reverse derivation and generalized reverse (θ, φ)- derivation on ideal, Lie ideal and Jordan ideal of prime, δ-prime and δ- semiprime rings. This work gives the generalization of many previous theorems that contributed to various branches of mathematics, such as extending Posner's theorem (1957) and finding the action (θ, φ)-derivation on a prime rings in Ashraf (2005). Being the derivations are important tools in ring theory, the results on derivations are one of the sources for developing of other theories, such as the theory of Hopf algebra action on rings, the theory of differential identities, the differential Galois Theory and others.

1.8 Tentative Research Chapters and Status

In this part of thesis, the selected objects that will be studied and discussed will be showed, for that, the tentative research chapters and the structure of this thesis are listed in Table 1.2.

Table 1.2

Tentative Chapters

Chapters	**Structure of the thesis**
1. Introduction	The introduction, research background, problem statements, research objectives, the scope of the study and significance of findings are presented in this chapter.
2. Literature Review	This chapter contains some definitions, examples and concepts of the rings and some results, which have been gotten about derivations on prime and semiprime rings.
3. Research Methodology	This chapter includes the research design and the research procedures and methods.
4. Derivation on Lie and Jordan ideals of a prime ring	This chapter study the left (θ, φ)-derivations, which is acting as a homomorphism on Lie and Jordan ideals of a prime ring.

(continue)

Table 1.2 *(continue)*

Chapters	Structure of the thesis
5. The commutativity of prime ring for generalized derivation on prime ring	This chapter study the generalized derivations on Jordan ideals of a prime ring.
6. The centralizing derivation on δ-prime ring	In this chapter, the centralizing derivation on δ-ideal of a δ- prime ring will be applied.
7. The centralizing reverse derivation on δ-prime rings	In this chapter, the centralizing reverse derivation on δ- ideal of a δ- prime ring will be applied.
8. Generalized reverse derivation in δ- prime rings and δ- semiprime rings	In this chapter, the generalized reverse derivation, related to a reverse derivation on δ- prime ring and δ- semiprime ring, and generalized reverse (θ, φ)-derivations, affined to a reverse (θ, φ)-derivations on δ- prime ring and δ- semiprime ring will be extended.
9. Conclusion	In this chapter contains summary about the research in general.