ROUTING AND THEORETICAL PROPERTIES OF OPTIMISED DEGREE SIX 3-MODIFIED CHORDAL RINGS FOR LARGE INTERCONNECTION NETWORK TOPOLOGIES

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ABSTRACT

This study aimed to develop a new degree six modified chordal ring, the optimised degree six 3-modified chordal ring, $CHR6o_3$ as a seed topology for large networks which is able to improve on the performance of existing degree six chordal rings by previous researchers. Its graph theoretical properties were analysed and its routing algorithm was proposed. The performance parameters of optimal diameter and optimal average path length in $CHR6o_3$ were first analysed theoretically by the number of nodes in its tree visualisation. Formulae for these two performance parameters were then generated, which enabled their comparison with those of degree six chordal rings by previous researchers for network sizes from 1200 to 12000 nodes. Graph theoretical properties encompassing asymmetry, existence of Hamiltonian and Eulerian Circuits, bounds for chromatic numbers, and conditions for different chromatic indices in $CHR6o_3$ were investigated. A suitable geometrical representation was constructed to illustrate its connectivity. An optimum free-table routing algorithm was developed and ran as a computer simulation to determine the shortest paths through which a message can travel through a network of $CHR6o_3$. $CHR6o_3$ was shown to have better performance in ranges of large networks compared to degree six chordal ring topologies proposed by preceding researchers, with diameter 8 at 12000 nodes. The results from the formulations of $CHR6o_3$ were validated by comparing them to those from the computer simulation. Theorems regarding aforementioned graph theoretical properties were developed and successfully proven. A geometrical representation based on snowflakes was also developed to illustrate the connectivity in a $CHR6o_3$ network. In conclusion, this research succeeded in improving over the performance of existing degree six chordal ring topologies proposed by preceding researchers, based on diameter and average path length in large networks. This implies that $CHR6o_3$ is a seed topology that can be considered as a multiprocessor interconnection network.

PENGHALAAN DAN SIFAT-SIFAT TEORI CINCIN KORDAL 3-DIUBAHSUAI DARJAH ENAM YANG DIOPTIMUM BAGI TOPOLOGI RANGKAIAN PENYAMBUNGAN BESAR

ABSTRAK

Kajian ini bertujuan membangunkan suatu cincin kordal baru, cincin kordal 3-diubahsuai darjah enam yang dioptimumkan, $CHR6o_3$ sebagai topologi permulaan untuk rangkaian besar yang mampu menambahbaik prestasi cincin kordal darjah enam sedia ada yang telah dikaji oleh penyelidik-penyelidik sebelumnya. Sifat-sifat grafnya juga dianalisis secara teori dan algoritma penghalaannya dicadangkan. Parameter prestasi garis pusat optimum dan panjang laluan purata optimum $CHR6o_3$ mula-mulanya dianalisis secara teori melalui jumlah nod dalam visualisasi pokoknya. Formula untuk kedua-dua parameter prestasi tersebut kemudiannya dijana, bagi membolehkan perbandingannya dengan cincin kordal darjah enam oleh penyelidik-penyelidik sebelumnya untuk saiz rangkaian 1200-12000 nod. Sifat-sifat graf secara teori yang disiasat merangkumi asimetri, kewujudan Litar Hamilton dan Litar Euler, batasan untuk nombor kromatik, dan syarat-syarat bagi indeks kromatik berbeza dalam $CHR6o_3$. Perwakilan geometri yang bersesuaian dibina untuk menunjukkan penyambungan di dalam $CHR6o_3$. Algoritma penghalaan bebas-jadual optimum dibangunkan dan dilaksanakan sebagai simulasi komputer untuk menentukan laluan terpendek di mana mesej boleh dihantar dalam sesuatu rangkaian $CHR6o_3$. $CHR6o_3$ ditunjukkan memberi prestasi yang lebih baik dalam julat rangkaian besar dengan garis pusat 8 pada 12000 nod, berbanding topologi cincin kordal darjah enam yang dibangunkan oleh penyelidik-penyelidik sebelum ini. Keputusan daripada formulasi $CHR6o_3$ disahkan dengan membandingkannya dengan keputusan daripada simulasi komputer. Teorem-teorem berkaitan dengan sifat-sifat graf secara teori yang dibina telah berjaya dibuktikan. Perwakilan geometri berasaskan emping salji juga telah dibina untuk menggambarkan penyambungan dalam sesuatu rangkaian $CHR6o_3$. Kesimpulannya, kajian ini telah berjaya menambahbaik ke atas prestasi cincin kordal darjah enam sedia ada yang dibangunkan oleh penyelidik-penyelidik sebelum ini, berdasarkan garis pusatnya dan panjang laluan puratanya dalam rangkaian yang besar. Implikasinya, $CHR6o_3$ ialah suatu topologi permulaan yang boleh dipertimbangkan sebagai satu rangkaian penyambungan multiprosesor.

CONTENTS

CHAPTER 4 THE OPTIMISED DEGREE SIX 3-MODIFIED CHORDAL RING

CHAPTER 5 OPTIMUM FREE-TABLE ROUTING ALGORITHMS

LIST OF TABLES

Table No. **Page**

05-4506832 pustaka.upsi.edu.my Perpustakaan Tuanku Bainun Kampus Sultan Abdul Jalil Shah PustakaTBainun ptbupsi

LIST OF FIGURES

LIST OF SYMBOLS

d	Layer number in tree visualisation
d_{avg}	Average path length
d_{avo}	Optimal average path length
d_{min}	Minimum path length
$\deg G$	Graph degree
$D(G)$	Diameter
E	Edge set
gcd	Greatest common divisor
G	Graph
h_i	Chord
K_n	Complete graph of n vertices
$\log_{10}$	Logarithm to the base 10
max	Maximum value
$mod\ N$	Modulo operation N
N	Number of nodes
N_d	Number of nodes in layer$-d$ of the tree visualization
N_{do}	Number of nodes in the optimal graph
N_i	Node
Q	Set of chords
s	Ring link
V	Vertex set
$\sum$	Summation

APPENDIX LIST

CHAPTER 1

INTRODUCTION

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1.1 Parallel and Distributed Computing

The increasing demand for higher computing power for both scientific research applications as well as domestic uses have increased the necessity for computer architectures to be constructed based on concepts of parallel and distributed computing. These networks are able to reduce the time taken for complex computations as multiple processor units are arranged to sequentially work on a problem instead of one. In a parallel network, there are multiple memory units shared by multiple processor units which work simultaneously. These units are connected by an interconnection network topology. In distributive computing, the concept is similar, but all memory units are distributed evenly to all processor units, such that each processor unit has its own memory unit. For both cases, the interconnection

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network topology determines the overall performance of the whole network (Kotsis, 1992). Hence, optimised interconnection network topology planning has become more and more important in assuring system efficiency.

1.1.1 Network Topology Models

One application of Graph Theory is that a graph, G can be used to model a physical network topology. Reference material involving Graph Theory or Discrete Mathematics, such as that of Rosen (2012) will define a graph as $G(V, E)$ where V is the set of vertices which are connected by the edges in the set of edges, E. A network topology is similar in the sense that vertices represent the processor units in the topology which are connected by links, represented by the edges of the graph. The general objective of this graph model was to provide a visual representation or a framework that could be analysed in the process of planning the network before actually constructing it in real life. Among the important parameters that can be analysed from this model are performance, robustness, and cost.

One such model used in network topology planning is the circulant graph. A generalised definition such as that of the one provided by Kotsis (1992), defines a circulant graph $CG(N, h)$ as an undirected graph of N vertices where any $i-$th vertex is adjacent to the $(i+j)-$th and $(i-j)-$th vertices for each j in the set of edges, h. Examples of circulant graphs are cycles, complete graphs, and chordal rings, the latter being focused on in this thesis. Notable properties of the circulant graph for interconnection network topologies are its connectivity, symmetry, and fault tolerance.

1.1.2 Communication in Network Topologies

A message travels from a source node to its destination node through an interconnection network topology in two different ways. In the first case, a destination node is adjacent to its source node. The message travels directly to it through a single link. In the second case, the destination node is not adjacent to its source node. For this case, a message needs to travel through a number of intermediate nodes to reach its destination node. The number of nodes travelled through is usually referred to as the network diameter in undirected cases.

However, for optimum performance, a message should travel through the least number of intermediate nodes to reach its destination node. Such a problem can be solved by routing, which is also be modelled by using Graph Theory as a shortest-path problem. Routing is generally defined as the determination of the most efficient paths for a message to travel within an interconnection network topology. The path taken by a message needs to be as short as possible because large messages tend to reduce node throughput, the rate at which messages can be processed, thereby increasing system latency.

1.1.3 The Motivation of a Network Topology

Although it is possible to design a network topology by eye, desired results may not always be produced. Bottlenecks at certain nodes may occur very frequently, increasing node congestion and latency, slowing down performance. The overall connectivity between all processor nodes may not be high as well; there may be few or only one path between a particular source node and destination node, such as the case of the bus topology. In this case, the robustness of the network is low, and a single link failure may disconnect the entire network. Network link failures are caused by wear and tear over time or damage from human error. These may impair the performance of the overall network or even cause it to cease functioning.

Since network topologies can be modelled after a graph, Graph Theory has become an invaluable resource in network planning. By analysing graph theoretical properties such as symmetry, connectivity, diameter, average path length, Hamiltonicity, and Eulerity of a certain proposed network topology modelled after a graph, its performance in terms of congestion and latency can be determined more systematically without the need of trial and error or brute force testing. These properties, especially that of symmetry, also provide the information required to develop better and more efficient routing algorithms. Incorporating a redundancy into the network interconnection model during planning will maintain connectivity in the case of multiple link failures, allowing the network to continue functioning with a certain degree of efficiency until repairs can be carried out.

1.2 Loops, Double Loops and Triple Loops

The first implemented solution for increasing reliability in a network was the ring topology, modelled by a cycle graph. The advantages of the ring were its low complexity and simple routing scheme, as well as the redundancy that a message could reach is destination node by moving in either direction through the ring, allowing it to stay connected if one link fails (Dubalski et. al, 2012). This redundancy, as well as the connection of the final node back to its initial node, defined the ring topology as a loop network.

However, it quickly became obvious that more than one link failure could disconnect the network. The ring topology also had very poor communication speed since messages had to travel through multiple nodes to reach their destination node, increasing latency. Network topologies modelled based on complete graphs were also proposed. These had high communication performance and high fault tolerance, but were costs prohibitive and required high processing power, and hence could only be implemented in small networks.

In 1981, Arden and Lee proposed the degree three chordal ring, an interconnection network topology that was essentially 'halfway' between the ring and complete graph topologies. Later researchers, Browne and Hodgson (1990) proposed the first degree four chordal ring, a double loop network. A triple loop network, the degree six chordal ring, was proposed by Matroud in 2006. Multiple loop interconnection network topologies confer to higher connectivity and robustness, since there are multiple path options, both clockwise and anticlockwise, for a message to travel through the network.

1.3 Routing Algorithms

Routing algorithms are defined as the set of rules in the process of selecting best paths for a message from a source node to reach all its destination nodes in a network. Various routing algorithms for chordal rings have been proposed by previous researchers, such as compact routing (Narayannan & Opatrny, 1999), optical routing (Narayannan et. al, 2001), topological routing (Gutierrez et. al, 2009), and optimum free-table routing (Farah et. al, 2010a). In general, a routing algorithm for chordal rings has the following steps:

1. Finding the acceptable periods for N.
2. Finding the available chord lengths for each period.
3. Calculating diameter.
4. Selecting a set of chord lengths which give the smallest diameter.

A good routing algorithm will ensure the best performance of a network since messages reach their destination nodes through the shortest possible paths. Further properties of a good routing algorithm are reliability and being error-free. Again, to develop these algorithms, it is important to know the graph theoretical properties of the network such as diameter and symmetry.

Another important part of a routing algorithm is its complexity. The complexity, or time complexity, of a routing algorithm is defined by Rosen (2012) to be a quantification of the amount of time taken to run as a function of the input. It is usually measured in the number of elementary steps involved in the routing algorithm. Hence, low diameters as well as symmetrical properties are preferred as they reduce the complexity of the algorithm.

1.4 Problem Statement

Based on the literature reviewed about the existing degree six modified chordal ring topologies, it was found all degree six modified chordal ring topologies could only be implemented in networks with even numbers of processor nodes. It was also found after further review that no traditional and modified chordal ring topologies allowed for an odd number of nodes, with the exception of the degree four and degree six traditional chordal ring topologies, subjected to certain constraints. Despite having simpler extensibility and symmetry, the performance of these traditional chordal rings topologies pale in comparison to their modified counterparts, in terms of latency for instance. Furthermore, increasing the size of a network topology after it has been laid down is difficult. Hence, it would be easier and more cost effective to choose a good seed topology. The first problem statement was to investigate the possibility of proposing a degree six modified chordal ring topology as a seed topology for large interconnection networks which is able to incorporate an odd number of nodes while improving on the performance of existing degree six traditional and modified chordal rings by previous researchers. Since large networks were considered, overall connectivity and robustness should be high as possible, hence the choice to improve on degree six traditional and modified chordal rings.

Next, the graph theoretical properties of the proposed topology had to be analysed. This was a crucial step in understanding the strengths, weaknesses, properties, and performance parameters of the proposed topology. The development of a routing algorithm was also essential to illustrate the most efficient paths a message can travel through the topology.

1.5 Research Objectives

From the problem statements, the following research objectives can be stated as follows:

1. To propose a new degree six modified chordal ring topology, called the optimised degree six 3-modified chordal ring, $CHR6o_3$ for large networks which is able to accommodate an odd number of processors and at the same time improving on topology performance of other degree six chordal rings by previous researchers; as well as to analyse its graph theoretical properties.
2. To develop a routing algorithm for the shortest paths for a message from one source node to reach all the destination nodes based on the proposed model, as mentioned in the first research objective.

1.6 Scope

This research focused on proposing a new degree six modified chordal ring called the optimised degree six 3-modified chordal ring, denoted as $CHR6o_3$ for implementation in large networks. This chordal ring network topology was modelled after a graph, $G(V,E)$ where the set of vertices, V represented the set of processor nodes and the set of edges, E represented the communication links connecting the processor nodes. Hence, the scope of this model only encompassed multiprocessor interconnection. The range of the large networks studied in this research refers to 1200 to 12000 processor nodes.

Only the undirected case was considered because each 'edge' in the graph structure can be replaced with two links in opposite directions (Narayanan and Opatrny,1999), ensuring that the graph model is strongly connected. Existing degree six chordal ring topologies, both traditional and modified were mentioned for the sake of comparing theoretical performance parameters and properties. The theoretical performance parameters in this research refer to optimal diameter and optimal average path length between specific nodes in the chordal ring topology. These parameters were tested for large networks ranging from 1200 up to 12000 nodes, providing an illustration of latency in the proposed structure.

Theoretical properties encompassed connectivity, node symmetry, link symmetry, node colouring, and link colouring. Node and link colouring encompassed the boundaries for chromatic number and chromatic index respectively, and rules to achieve proper colourings. The properties investigated regarding connectivity were the Hamiltonicity and Eulerity of the proposed structure. The routing algorithm based on the proposed degree six modified chordal ring refers to only the most efficient paths for a message from a source node to reach all the other nodes in the topology, with the assumption that no node failure or link failure occurred.

1.7 Contributions of the Research

Based on the research objectives, the rationale of this research was that it hoped to succeed in proposing a new seed topology for large multiprocessor interconnection networks with reduced latency compared to existing degree six traditional and modified chordal rings. A good seed topology is important in network planning because it is not cost efficient to change or extend a topology after it is laid down. Despite the scope being closed to multiprocessor interconnection, the outline of the topology was still modelled by Graph Theory. Hence, it can be applied not only to multiprocessors, but also to other large networks, both wire and optic.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

An interconnection topology is the basis in designing any computer network on any scale. Since it governs how nodes are connected in a network, it defines network performance in many aspects, such as latency, robustness, ease of maintenance, and efficiency (Bujnowski et. al, 2011). Thus, along with the increasing demand of higher and more robust computing power, it is important to choose a suitable physical and logical topology in the planning stage itself (Azura et. al, 2010), as changing the topology midway is impractical and incurs a high price (Bujnowski et. al, 2011).