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NUMERICAL STUDY ON SOME ITERATIVE METHODS FOR SOLVING NONLINEAR EQUATIONS BY USING SCILAB PROGRAMMING

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AHMED HADI MOHAMMED ABOAMEMAH

UNIVERSITI PENDIDIKAN SULTAN IDRIS
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DISSERTATION PRESENTED TO QUALIFY FOR A MASTER'S DEGREE IN EDUCATION (APPLIED MATHEMATICS) (RESEARCH AND COURSEWORK MODE)

FACULTY OF SCIENCE AND MATHEMATICS
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ABSTRACT

This research aimed to investigate the most efficient iterative method in solving scalar nonlinear equations. There are three iterative methods that are used to solve the nonlinear scalar equations that are Bisection, Secant and Newton Raphson's methods. These three iterative methods have different order of convergence. Bisection method is linearly convergence while Secant method is super linear and Newton-Raphson method has quadratic convergence. It is well known that the method that has a higher order of convergence, will perform much faster than others. Seven nonlinear scalar equations are considered based on the combinations of two or three functions and are solved by the Bisection, Secant and Newton-Raphson methods using Scilab programming language. The tolerance used is 10^{-10} and the performances of these methods are based on number of function evaluation, number of iterations, and computational or CPU time. Based on the numerical results of the seven nonlinear equations, it is observed that Newton-Raphson method is still the most efficient method but not for all the equations. Bisection method has fixed performances on all the nonlinear equations however, the method failed to converge for the imaginary root. On the other hand, the performance of Secant method is almost similar to Newton-Raphson method except for the nonlinear Equations (4.4), and (4.5) on the interval $[1.3, 2]$ and $[0, 1]$ respectively. In conclusion, Newton-Raphson method remains the best but not for all nonlinear equations since there are realistic circumstances that makes Newton-Raphson converges either slower or identical to Secant method. It is also proven that Secant method can perform faster than Newton-Raphson method depending on the form of the curve functions that corresponds to the approximate values. As implications, more than three combinations of the functions can be investigated and also the research can be extended to system of nonlinear equations.

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KAJIAN NUMERIKAL MENGGUNAKAN KAEDAH-KAEDAH ITERATIF UNTUK MENYELESAIKAN PERSAMAAN BUKAN LINEAR MENGGUNAKAN PENGATURCARAAN SCILAB

ABSTRAK

Kajian ini bertujuan untuk mengkaji kaedah iteratif yang paling efisen dalam menyelesaikan persamaan skalar bukan linear. Terdapat tiga kaedah iteratif yang digunakan untuk menyelesaikan persamaan skalar bukan linear iaitu kaedah Bisection, Secant dan Newton-Raphson. Ketiga-tiga kaedah iteratif ini mempunyai susunan penumpuan yang berbeza. Kaedah Bisection mempunyai konvergensi linear manakala kaedah Secant mempunyai konvergensi linear super serta kaedah Newton-Raphson mempunyai konvergensi kuadratik. Adalah diketahui bahawa kaedah yang mempunyai susunan penumpuan yang lebih tinggi, akan lebih cepat menumpu daripada yang lain. Tujuh persamaan skalar bukan linear telah dipilih berdasarkan kombinasi dua atau tiga fungsi dan diselesaikan menggunakan kaedah Bisection, Secant dan Newton-Raphson melalui penggunaan pengaturcaraan Scilab. Toleransi yang digunakan ialah 10^{-10} dan persembahan kaedah ini adalah berdasarkan kepada bilangan penilaian fungsi, bilangan lelaran, dan masa pengiraan atau CPU. Berdasarkan keputusan masalah berangka tujuh persamaan bukan linear, didapati bahawa kaedah Newton-Raphson masih merupakan kaedah yang paling efisien tetapi tidak untuk semua persamaan. Kaedah Bisection walaupun menunjukkan prestasi yang tetap bagi semua persamaan bukan linear namun, kaedah ini gagal untuk menumpu bagi nilai khayalan. Sebaliknya, prestasi kaedah Secant hampir sama dengan kaedah Newton-Raphson kecuali untuk persamaan bukan linear (4.4), dan (4.5) pada selang $[1.3,2]$ dan $[0,1]$. Kesimpulannya, kaedah Newton-Raphson kekal sebagai kaedah yang terbaik tetapi bukan untuk semua persamaan bukan linear kerana terdapat keadaan realistik yang menyebabkan penumpuan keadah Newton-Raphson sama ada lebih lambat ataupun sama dengan kaedah Secant. Ia juga terbukti bahawa kaedah Secant boleh menyelesaikan persamaan bukan linear lebih cepat daripada kaedah Newton-Raphson bergantung kepada bentuk fungsi lengkung yang sepadan dengan nilai anggaran. Sebagai implikasi, lebih daripada tiga kombinasi fungsi dapat diselidik dan juga penyelidikan dapat diperluaskan ke sistem persamaan bukan linear.

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CHAPTER 1

INTRODUCTION

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This chapter gives general conception about the research, such as the main concepts that explain the main path of the research, background of research, problem statement, objectives of research, research questions, significant of research, and chapter summary.

1.1 Background of Study

Solving nonlinear equations $f(x) = 0$ and system of nonlinear equations $F(X) = 0$ are one of the most important problems involved in solving science and engineering problems. Solving scalar and system of nonlinear equations means finding the values that satisfy them such that to find the zeros of nonlinear equations. When $f(x) = 0$ it means

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that the roots of the solutions can be found (Hasan, 2016). Often-times solving nonlinear equations are not available by usual algebraic analytic methods since it is difficult and take a longer time to solve. From this point, nonlinear equations need to be solved using numerical approximation methods depending on iterative processes to obtain an approximate solution. Numerical methods are used to find the approximate solution of nonlinear equations. Generally the most suitable methods to find the solutions of nonlinear equations that can not be solved analytically are the iterative methods which are classified into two categories: bracketing iterative methods and open iterative methods according to the requirements.

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The bracketing methods require an initial guess interval such that the function $f(x)$ has opposite signs at limits of the guess interval, whereas the open methods are different in requirement, some of them require one initial guess value while some others require two or more than two initial guess values (Intep, 2018). The popular methods for solving nonlinear scalar equations are Bisection method, Newton-Raphson method, Secant method, False Position method, Fixed point Iteration method, and Muller method. On the other hand, the methods to solve system of nonlinear equations are Fixed Point iteration method, Newton-Raphson method, Secant or Broyden's method and many more.

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The most important thing in the iterative methods is the difference in rate and order of convergence when finding the root of equation such that there is a difference in speed of convergence according to used iterative method. The method that has a high order of convergence, will perform much faster than others. Order of convergence of some iterative methods are linear, super-linear, and some others are quadratic and so on. Based on previous studies, Newton-Raphson method may converges faster than others according to its convergence order, but when comparison is made on the performance of iterative methods, there is a need to take into consideration many things. The importance ones are cost, the accuracy of the solution and speed of convergence Hasan (2016), Kumar and Vipan (2015). An iterative method that converges rapidly may takes longer time than an iterative method that converges more slowly but takes shorter time per-iteration. Based on that, it can be considered that the cost of an iteration is controlled by the required number of function evaluations per-iteration whereby the number of function evaluations is a good scale of cost (Ehiwario and Aghamie, 2014).

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Despite of the iterative methods to solve nonlinear equations, solving nonlinear equations itself is not an easy matter especially for combine nonlinear equations or higher degree of polynomials. But after the discovery of computers and software programmings, numerical methods became in the forefront of science in terms of importance and development. Therefore, the numerical methods found their way into

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scientific life not only as a necessary subject but as a development techniques to overcome many complex problems in scientific applications, including engineering and mathematical. The software helps to understand, and programming the iterative methods on the computer. Scilab programming is one of the software that can be used to introduce the iterative methods.

1.2 Preliminary Concepts

1.2.1 Fixed point

Definition 1 *(Burden et al., 2014)*

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Let x^* be any number in domain of the function, then x is a fixed point if image of x is itself x^* when the function is mapped.

As a mathematical definition: x^* is a fixed point for the function f if $f(x^*) = x^*$.

Fixed point problems are equivalent kinds in the following concepts:

1. Functions of g can be defined with fixed point x^* by rewrite the given function $f(x) = 0$ in many ways, for example $g(x) = x - f(x)$ or $g(x) = x - 2f(x)$.

2. Contrariwise, if x^* is a fixed point for the function g, then x^* is a solution for the function that defined by $f(x) = x - g(x)$.

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Theorem 1 *(Henrici, 1964)*

Let $g(x)$ be a continuous and differentiable function over the interval $I = [a,b]$ and $g(x) \in I, \forall x \in I$, in addition its first derivative g' exists on (a,b) and satisfies the following condition: $|g'(x)| \leqslant L$ for all $x \in (a,b)$, where L is a positive real number $(L \in (0,1))$ then g has one fixed point in I.

1.2.2 Convergence

Definition 2 *(Ehiwario and Aghamie, 2014)*

All iterative methods generate a sequence $\{x_n\}$. Then the generated sequence is said to

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converge to the accurate root x if $\lim_{n\to\infty} |x - x_0| = 0$.

Let $f(x) = 0$ be any nonlinear equation, then the equation can be written as $x = g(x)$. If x_0 forms the random starting point for the method, then the solution x^* of the equation $f(x) = 0$, $x^* = g(x^*)$ it can be found by the following numerical sequence:

$$x_{n+1} = g(x_n), \text{where} \quad n = 0, 1, 2, 3, \cdots . \tag{1.1}$$

The iteration above called a Picard process and x^* the limit of the sequence, is called the fixed iterative method.

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In order to the sequence above tend towards the root of equation, it has to be convergent. A sufficient condition for convergence is the following theorem:

Theorem 2 *(Burden et al., 2014)*

Let $x = g(x)$ has a solution x^* within the interval

$$M = [x^* - a, x^* + a] = \{x : |x - x^*| \leq a\},$$

and let $g(x)$ satisfies Lipschitzs condition.

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$$\textit{Exists } L < 1: \quad \textit{for all } x \in M, \quad |g(x) - g(x^*)| \leq L|x - x^*|.$$

Then for each $x_0 \in M$: the solution will be unique in the interval M and all iterated values x_n belong to M and converge towards the solution x^*.

For Lipschitz's condition to be satisfied it is sufficient that for every $x \in M$, $g'(x)$ exists and is such that $|g'(x)| \leq m$ where $m > 1$.

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1.2.3 Taylors Expansion Theorem

Suppose that the function f is a continuous and $(n+1)$ times differentiable over the interval $[a,b]$ and $x_0 \in [a,b]$. Then for every $x \in [a,b]$, there exists a number $\xi(x)$ between x_0 and x with $f(x) = P_n(x) + R_n(x)$, where

$$\begin{aligned} P_n(x) =& f(x_0) + f'(x_0)(x-x_0) + \frac{f^{(2)}(x_0)}{2!}(x-x_0)^2 + \frac{f^{(3)}(x_0)}{3!}(x-x_0)^3 + \cdots \\ &+ \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n \end{aligned}$$

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and

$$R_n(x) = \frac{f^{(n+1)}(\xi(x))}{(n+1)!}(x-x_0)^{n+1}$$

Hither $P_n(x)$ is called the nth Taylor polynomial for f about x_0, and $R_n(x)$ is called the truncation error associated with $P_n(x)$ (Burden et al., 2014).

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1.2.4 Compute Error

Numerical methods are used to solve complex problems approximately whose its solutions are not available by algebraic methods. Numerical analysis works on finding the approximated solutions for complex problems and calibrating the error that arises from using the numerical methods. There are two types of errors. The first one is the absolute error that equals the difference between the exact value and the approximate value.The second one is relative error that equals the ratio of the absolute error to size of the exact solution (Shanker, 2006).

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If x represents the true value or exact solution and x^* represents the approximate value, then.

$$Absolute\ error = |exact - approximate| = |x - x^*|, \quad \text{and}$$

$$Relative\ error = \frac{|exact - approximate|}{exact} = \frac{|x - x^*|}{|x|}.$$

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1.2.5 Order and Rate of Convergence

If the sequence $x_0, x_1, x_2, ..., x_n$ converges to the root of equation $f(x)$ generated by an iterative method, then the most important thing in choosing the iterative method is convergence and at the same level of important is order or speed of convergence for that convergent method, which is measure how fast the sequence converges to solution.

Now how to determine the order and rate of convergence for any iterative method, according to Hasan and Ahmad (2015), the sequence of approximations $x_0, x_1, x_2, ..., x_n$ to the solution x obtained by any iterative method. Thus, the order and rate of convergence of the above sequence that converges to x can be determined if there is a number p and

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a positive constant C such that:

$$\lim_{n\to\infty} \frac{|x_{n+1} - x|}{|x_n - x|^p} = C. \tag{1.2}$$

Then, the convergence order of the sequence x_n is p and with rate or asymptotic error constant C which is usually rely on derivatives of function. If, $p = 1$ then the sequence x_n converges linearly to the solution x i.e. the order of convergence is linear. And if, $p = 2$ then the sequence x_n converges quadratically to x, i.e. the order of convergence is quadratic. Thus, if $p = n$, then the order of convergence is n.

The above formula can be written as follows:

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$$\lim_{n \to \infty} \frac{|e_{n+1}|}{|e_n|^p} = C, \tag{1.3}$$

where e_n and e_{n+1} are the errors in nth and $(n+1)$th iterates respectively. i.e.

$e_{n+1} = x_{n+1} - x$ and $e_n = x_{n+1} - x$.

1.2.6 Intermediate Value Theorem

Definition 3 *(Chhabra, 2014)*

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The Intermediate Value Theorem for a real-valued function $f(x)$, continuous and defined for all x over a closed interval $[a,b]$, such that $f(a) \cdot f(b) < 0$ then the open interval (a,b) contains at least one point c for $f(c) = 0$, where c is one of the roots of the function f .

If $f(a) \cdot f(b) > 0$ then that means there are two possible results, the first one is, there is no any root in the interval (a,b) and the second one is, there are even number of roots in the interval (a,b).

The Intermediate Value Theorem is important for identifying zeros of functions and bracketing iterative methods. In order to determine the intervals that contain the

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roots of a function need to set up a table of values of $f(x)$ for different values of x. If the signs of the function $f(x)$ changes at two successive values of x, then there is at least one root lies between that two successive values (Jain, 2003).

1.3 Problem Statement

There are many basic iterative methods and others that have been proposed and developed for solving scalar and system of nonlinear equations and there are also different types of nonlinear equations (algebraic, trigonometric, logarithmic, and exponential equations). These iterative methods are different in their efficiency according to their speed of convergences and cost in addition since they do not have all the same conduct. So selecting and identifying the most appropriate iterative method for solving scalar and system of nonlinear equations is not an easy matter since there are many things that have to take into account. Firstly the order and rate of convergence, and secondly is the efficiency of iterative methods which is take into account the order of convergence with the required number of functions evaluations per iteration, and ensure convergence such that whether the method guaranteed convergence for the given equation. According to results of studies of Hasan (2016) and Ehiwario and Aghamie (2014), there are difference in the performances of iterative methods, where the first study shows that Secant method converges to the exact solution at the 5th iteration while Newton-Raphson

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method converges at the 4th iteration and it is the fastest. The latter study shows that the Secant method is the fastest and it converges to the exact root at the 6th iteration, whereas Newton-Raphson method is slower than Secant method, where it converges to the exact solution at the 8th iteration. Hence there is no definite answers on which method is the fastest. Is there specific cases in which Secant method can perform faster than Newton's method?. Therefore, this research is important to determine the efficiency of some iterative methods in solving some nonlinear equations.

Although based on the previous studies, Newton-Rapson method is faster than others iterative methods since the method is order 2, however certain question need to be investigated to make sure whether, Newton's method always the fastest. The questions are:

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1. Does Newton-Raphson method remain faster than other methods for all different equations?
2. Does it remain faster than other methods for all different root of given equation?

Another important aspect that should be considered is when the given scalar nonlinear equation has n roots. As an example how to find all roots if the function approximated using Newton-Raphson or Secant methods.

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In general it is important to identify which one of this iterative methods is the most efficient and suitable for solving nonlinear equations.

1.4 Objectives of Research

The main goal of this research is to determine the best iterative method for solving nonlinear equations through investigating the efficiency of basic iterative methods numerically using **Scilab programming** language. Hence, the objectives of this research are as follows:

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1. To solve 7 different nonlinear scalar equations by Bisection, Secant and Newton methods to compare their performance using Scilab programming.
2. To determine the fastest iterative method that reaches the root with least time, lowest iterations number and lowest function evaluations.
3. To determine the cases or the situations in which Secant method can out perform Newtons method.

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1.5 Research Questions

1. What is the best iterative method that cost least time, lowest iterations and lowest functions evaluations to reach the desired root?

2. Does the performance of the basic iterative methods remain constant for all different equations?

3. Do the iterative methods have similar behaviour for all different kind of nonlinear equations?

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4. What are the situations in which secant method can perform faster than Newton's method?

1.6 Significant of Research

Solving nonlinear equations is one of the most important computational problems. It appears in a large diversity practical applications in engineering, physics, chemistry, bio-science and many other areas of study. For that reason the need to investigate in the efficiency of basic iterative methods in solving nonlinear equations is important and

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this research looks forward to contribute. Hence, this research is important

1. To identify and select the most appropriate iterative method to solve nonlinear equations.

2. To determine a strategy to find all different roots of nonlinear equations.

3. To determine number of functions evaluations required to solve nonlinear equations by the iterative methods.

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4. To determine whether there is one guaranteed alternative iterative methods that can overcome the divergence.

1.7 Limitation of Study

The major goal of this research is to identify the best iterative method for solving scalar nonlinear equations through investigating the efficiency of some iterative methods. The iterative methods that will be studied to investigate the efficiency for solving **scalar nonlinear equations** are Bisection, Secant and Newton-Raphson methods.

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These methods are tested on 7 different types of scalar nonlinear equations. This research is limited to scalar nonlinear equations because it is important to study on scalar nonlinear equations before focusing on system of nonlinear equations. This research is limited to only 7 types of nonlinear equations because these 7 equations can be classified into several different categories of the combinations of exponential, trigonometric, logarithmic, and algebraic functions. This research is also limited to cubic functions. Although only 7 equations are included in this dissertation, the author had actually tried many other equations and for out some of the equations, gives similar results with the 7 equations.

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1.8 Scope of Study

The nonlinear equations can be divided into two major classes: nonlinear scalar equations and system of n nonlinear equations in n variables. There are three iterative methods that can be used to solve scalar nonlinear equations such as Bisection method, Secant method and Newton's method (Ehiwario and Aghamie, 2014). This research addresses numerical study on some iterative methods for solving nonlinear equations by using Scilab programming language. It focuses on solving nonlinear scalar equations. Nonlinear scalar equations are chosen because most real life applications problems are nonlinear equations. Scilab programming language is used in this research because ac-

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cording to Salleh et al. (2012) it is considered one of the powerful programmings and proper tool to use in solving mathematics problems that need to be solved by numerical methods which are one of the algorithms that involved in computational programming.

1.9 Chapter Summary

This dissertation have five chapters. Chapter One gives the introduction of the iterative methods for solving scalar and system of nonlinear equations. Chapter One also discuses the objectives, research problem statement, research questions, limitation and scope of study. In Chapter Two, the previous studies on some iterative methods are explained in detailed. Chapter Three gives the detailed explanation on all the methods used in this dissertation such as the Bisection, Secant and Newton-Raphson methods. Chapter Four gives the numerical results for 7 nonlinear scalar equations. Lastly, Chapter Five concludes the dissertation by giving the answer to the research questions.

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