ESE Problem Solving model in solving mathematical word problems: A preliminary study

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ABSTRACT

This study is to examine the impacts of a brand-new innovative ESE Problem Solving Model which theoretical based on the Polya's (1945) Model problem solving to teach primary pupils tackling the mathematical word problems. There are three systematic and straightforward steps which are extract, synthesis and execute. The researchers employed a quasi-experimental research design in one of the national primary school in Sabah, Malaysia. A convenience sampling technique was used to collect a sample of 31 primary five pupils. statistical package for the social sciences (SPSS) version 26 was used for data entry and data analysis. The results of this study showed that the ESE Problem Solving Model is one of the potentially practical models to help pupils to solve mathematical word problems.

Keyword: *mathematics, problem-solving, Primary mathematics, learning mathematics, innovation*

ABSTRAK

Kajian ini bertujuan bagi menentu keberkesanan Model Penyelesaian Masalah ESE yang inovatif dan baharu. Model Penyelesaian Masalah ESE berasaskan teori penyelesaian masalah Model Polya (1945) mendedahkan murid sekolah rendah bagi menyelesaikan masalah ayat matematik. Terdapat tiga langkah sistematik dan mudah iaitu ekstrak, sintesis dan laksanakan. Reka bentuk kuasi eksperimen dan teknik persampelan secara kebetulan digunakan dalam kajian ini. Sampel terdiri daripada 31 orang murid tahun lima di sebuah sekolah rendah kebangsaan di Sabah, Malaysia. SPSS versi 26 digunakan untuk menganalisis data. Hasil kajian ini menunjukkan bahawa Model Penyelesaian Masalah ESE adalah salah satu model yang praktikal untuk membantu murid menyelesaikan masalah ayat matematik.

Kata kunci: *matematik, penyelesaian masalah, Matematik sekolah rendah, pembelajaran matematik, inovasi*

INTRODUCTION

Problem-solving is the core of learning mathematics (Siti Rahaimah Ali, 2019). The ultimate objective of Malaysia primary mathematics curriculum is developing pupils' mathematical knowledge and skills in problem-solving and adjust various strategies in solving problems (Malaysia Ministry of Education, 2018). Humans learn numeracy skills (number sense, pre number), arithmetic skills (four basic

operations), mathematical skills (measurement, estimations) and transform words problems to mathematical problems and so on to solve our daily problems.

Mathematics problem solving may help pupils to gain a deeper understanding of both conceptual understanding of mathematics topics and the application of procedural computational skills (Silver, 2013). For example, a question about 4 + 3, pupils may demonstrate their procedural fluency by computing step by step the total amount to get the answer of 7. When the pupils can perform the computation procedure, they develop their computation skills (Silver, 2013). Conceptual understanding for addition means sums of those addend values. Pupils can demonstrate conceptual understanding using words, concrete materials, pictures, or numerals (Silver, 2013). When the pupils develop their understanding about addition, they may apply that understanding in different situations that involve additions (Silver, 2013).

A mathematical problem is not just an exercise, it is something new, at first pupils will feel lost, and then pupils need to work it by themselves with their existing mathematical knowledge and skills (Smoryński, 2020). There are two types of problems: routine problems and non-routine problems. A routine problem which refers to pupils needs to apply their mathematical knowledge and skills based on their previous learning experiences (Siti Rahaimah Ali, 2019). For example, Mary has four sweets, Nancy has 2, what is the sum? Pupils can solve the question by adding 4 and 2 and get the answer of 6. The non-routine problem needs pupils to comprehend and recall a variety of mathematical concepts, then choose and extend that conceptual understanding to build up a logical solution (Siti Rahaimah Ali, 2019). Take for an instance, how to measure 3 ℓ of water if only provide a 7 ℓ container and a 2 ℓ container? The pupils need to be brainstorming and develop some problem-solving strategies such as try and error or draw a diagram to solve the problem. For example, Pupils need to the full fill the 7L container with water, pour in 2 ℓ of water into the 2 ℓ container, pour out the 2 ℓ water from the 2 ℓ container then repeat the action then will get 3L of water. Further to describe that mathematical problem needs a solution and not just an answer. A solution included the reasoning and the articulation of logical steps that demonstrate the whole mathematical process (Smoryński, 2020).

However, Munirah (2016) pointed out that Malaysia primary five pupils still weak in mathematical problem-solving skills in mathematics subject. Tambychik and Meerah (2010) further indicated that many mathematics skills were involved in problem-solving such as number-fact, visual-spatial and information skills. Information skill was the most critical among Malaysia primary pupils. We observed that our pupils could read and understand the questions as well as performed Polya's Model first step - understand the problem. However, we found that most of them might ignore some essential parts of pre-solving problems skill which is extracting the vital elements from the question and give a proper answer to the questions with proper units. Therefore, we proposed that the ESE problem-solving model has a positive effect on helping pupils in solving mathematical word problems. The ESE Problem Solving Model aims to reduce pupils' careless mistake while solving mathematics problems and let the pupils explore the questions and solve the question step by step. Therefore, the objective of this paper is to:

1. Determine the impacts of ESE problem-solving model in helping primary year five pupils to solve mathematics word problems.

Literature Review

George Polya, a Hungarian mathematician who outlined the following four-step process for addressing problems: (1) Understand the problem (2) Devise a plan (3) Carry out the plan (4) Examine the result (Liljedahl et al., 2016). Polya (1945) pointed out that it is unwise to tackle a question that we do not understand. He further detailed it is a sorrow to work out the problem that we do not interest. Teachers should be aware of these unwise and sorrow things happen in their mathematics classroom (Polya, 1945). The student should understand the problem before answering it. We do agree that understanding is vital in mathematics problem-solving. However, Polya (1945) stressed that the teacher seldom would miss the questions: What is the unknown? What are the data? Moreover, what is the condition? To make

sure the pupils get more awareness of this gap, especially for those pupils who are weak in mathematics, we suggested that the first step of ESI problem solving model start with the "Extract", and it should be a proactive action that to extract the data given and the unknown.

Followed by the Polya' (1945) model second step, it requires pupils to devise a plan to solve the problem after knowing the connection between the data and the unknown. Germain-Williams (2017) listed out the problem-solving plans that might be used to solve new problems such as: act it out, analyse the units, convert to algebra (d) create a physical representation, use deductive logic, draw a diagram, look for a pattern, work backwards and so on. We proposed that "Synthesis" as step two in ESI problem-solving model. It is essential to synthesis the keywords and connects to the data given by the question. Pupils maybe will misinterpret the keyword of the problem. For example, Dylan has 2 sets of 3 colourful pencils, find the total of Dylan's pencils. Pupils may find the keyword is "total", but most probably, the pupils will directly use addition to find out the answer. However, the pupils should Synthesis the keywords and connect to the data given by the question that 2 sets of 3 colourful pencils, which means that the pupils need to choose the multiplication to solve this question instead of addition.

For the third step and the last step, Polya (1945) suggested that carry out the plan, and it is essential to examine the solution with reasons and verify of operations. With comparing with ESE problem-solving model, we proposed a similar step "execute" that combining the implementation of the procedure and operation and review the solution as the last step. The rational of the ESE model is to save time in answering mathematical problem solving. As pupils are taught to extract the main points of the question, synthesis the key words connect to the data given by the question and execute the computational procedure.

Table1: Difference between Polya's (1945) model and ESE problem-solving model

Polya's model (1945)	Gap	ESE problem-solving model (2019)
1. Understand the problem	How to understand? What are the outputs to show understanding?	1. Extract
2. Devise a plan	Before devising a plan, what should pupils do?	2. Synthesis
3. Carry out the plan	This step is similar	2. Execute
4. Looking backwards	Looking backwards is time-consuming	3. Combined with the third step

ESE problem-solving model

Figure 1: ESE problem-solving model

Figure 1 shows the flow of the proposed ESE problem-solving model. The purpose of this model is to guide pupils in solving mathematics word problems. Based on Figure 1, there are three basic steps in this model, namely Extract, Synthesis and Execute. These steps are similar to the model proposed by Polya (1945), but they have their characteristics. The first step of this model requires the pupils to extract the correct information from the questions. The pupils must express their understanding of the given question by their own words or graphics.

The second step is known as Synthesis. Pupils are required to Synthesis the core elements that need to be solved, which generally be stated in the last sentence. Besides, they should be aware of the units required for the answers. All the processes involved need to be constructed in a systematic order before applying it practically.

The third step is to Execute. Pupils will apply relevant strategy to cope with the problem (choose from the 4 basic operations, combine operations). The relevant strategy stated here might be the trial and error, draw a graphic, build a table, make an analogy, working backwards, make a simulation and so on.

Here is the example of implementing the ESE problem-solving model.
LOTS question (extract from UPSR year 2019 paper, Question 8(a))
Table 2 shows the volume of milk,ℓ in three containers, R, S and T.

Container	Volume
R	2.5 ℓ
S	0.75ℓ
T	3.07ℓ

a) **Calculate the total volume of milk, in ℓ in the three containers**

Extract	Synthesis	Execute
R= 2.5 ℓ	The total volume of liquid of	*2.50ℓ*
S= 0.75ℓ	3 containers	*0.75ℓ*
T= 3.07ℓ		+ *3.07ℓ*

		6.32 ℓ

HOTS question (extract from UPSR year 2019 paper, Question 6(b))
The price of a bicycle in a shop is RM180. The shop gives a 20% discount to Adam's father. Calculate the price paid by Adam's father.

Extract	Synthesis	Execute	
Price before discount = RM180	Price after discount (RM)	*Discount* *20% x RM 180 =* $\frac{20}{100}$ *x RM180* *=RM36*	*Price after discount* *RM 180* - *RM 36* ---------------- *RM 144*
Discount= 20%			

Extract the information given from long sentences into short sentences using mathematical symbols.
Synthesis the requirements of the question and aware of the units required in the question.
Execute the strategy and make sense of the answers.

METHODOLOGY

A one-group pretest−posttest quasi-experimental research design was conducted. The mathematics teacher was trained with the use of ESE problem-solving model in teaching mathematical problem for two days. The mathematics teacher taught only one group of her primary five pupils who participated

in the ESE problem-solving model for eight weeks. Convenient sampling methods were used to gather only one group data for the participation of 31 primary five pupils at one of the national primary schools in Sabah, Malaysia and there is no control group. The researchers included the participants that were available and voluntarily. This study intended to evaluate pupils' problem-solving performance before and after using the ESE problem-solving model. Wilcoxon signed-rank test was used to compare pretest and posttest results. Eventually, the researchers would examine the impact of ESE problem-solving model among the pupils and provide empirical data on whether they have significant differences in their problem-solving performance.

Finding and Discussion

31 Pupils who took part in this study were all 11 years old primary five pupils. There were 17 boys (54.8%) and 14 girls (45.2%). They were from one of the national primary school in Sabah, Malaysia. The descriptive statistics of the sampled pupils was tabulated in Table 2. The results show that the pupils scored a higher mean (m = 75.71) in the posttest compared to the pretest (m = 52.45). The minimum and the maximum score of the pretest was 32 and 86 while the minimum and the maximum score of the posttest was 52 and 93.

Table 2: Descriptive statistics of the sample's gender

	N	Mean	Std. Deviation	Minimum	Maximum
Pre	31	52.4516	14.75994	32.00	86.00
Post	31	75.7097	11.08511	52.00	93.00

Pupils were given the pretest before they explore the ESE problem-solving model, whilst the posttest was given after the pupils had been trained for 8 weeks to access the ESE problem-solving model. The pretest and posttest had the identical set of questions which were randomised to avoid the pupils memorise the answers. The results of the pretest and posttest are shown in Table 3. The results show that all 31 pupils scored positive ranks with 16 mean rank and sum of ranks was 496. There were no negative ranks and ties in these results.

Table 3: Wilcoxon Signed Ranks Test results

		N	Mean Rank	Sum of Ranks	Sum of Ranks
Posttest – Pretest	Negative Ranks	0[a]	.00	.00	.00
	Positive Ranks	31[b]	16.00	496.00	496.00
	Ties	0[c]			
	Total	31			

a. Posttest < Pretest

b. Posttest > Pretest

c. Posttest = Pretest

Additionally, analysis through a Wilcoxon signed-rank test showed that a 8 week, thrice weekly ESE problem-solving model did elicit a statistically significant change in 31 pupils with mathematical problem solving performance ($Z = -4.866$, $p = 0.000$). The analysis is a strong indication that the pupils have made remarkable progression in their learning outcomes after using the ESE problem-solving model to solve mathematical word problems.

Table 4: Test statistics[a] results

	Posttest-Pretest
Z	-4.866[b]
Asymp. Sig. (2-tailed)	.000

a. Wilcoxon Signed Ranks Test

b. Based on negative ranks.

Based on the pretest and posttest, the researchers found that the ESE problem solving model can enhance mathematical problem solving among pupils as well as to reduce their mistake in solving mathematical word problems.

CONCLUSION

The results of this study show that using ESE problem-solving model guide the pupils to explore the questions in actions (extract, Synthesis and execute) and reduce the careless mistake while solving mathematics problems. From the outcome of the pretest and posttest results, pupils reported being accustomed to applying what they learnt from the ESE problem-solving model in solving mathematical word problems. This result exemplifies that the pupils were able to accept what was being explored to the ESE problem-solving model.

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