MODELING THE SHAPE OF RED BLOOD CELL USING THE PDE METHOD

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ABSTRACT

This study aims to model the shape of a normal red blood cell (RBC) and two shapes of sickle cells using the partial differential equation (PDE). The development of this technique was based on the use of an elliptic PDE and a set of four periodic boundary conditions. The PDE method can generate surfaces of geometries from a small number of parameters. Furthermore, the shape of the surfaces generated by the PDE method is based on a boundary representation and can easily be modified since it is characterized by data distributed around the boundaries. In this study, the shapes of the generated PDE-based representation of a normal RBC and sickle cell has been sketched by using MATLAB program. The findings showed that the PDE method is suitable for representing the shape of a normal RBC and sickle cells. Besides that, the data regarding the radius and height from the normal RBC and one of the sickle cells, are then used to obtain four equations. These equations can be utilized for future prediction in designing both normal RBC and sickle cells. In conclusion, the PDE method can generate smooth parametric surface representations of any given shape of blood cells. The study implicates that the PDE method is capable of generating surfaces of complex geometries.

PERMODELAN BENTUK SEL DARAH MERAH MENGGUNAKAN KAEDAH PPS

ABSTRAK

Kajian ini bertujuan untuk membina model bentuk normal sel darah merah (SDM) dan dua bentuk sel sabit menggunakan persamaan pembezaan separa (PPS). Kaedah untuk memperkenalkan teknik ini adalah berasaskan kepada hasil penyelesaian PPS eliptik bersama empat syarat sempadan yang berkala. Kaedah PPS mampu menjana permukaan geometri dengan bilangan parameter yang sedikit. Tambahan pula, permukaan bentuk yang dijana oleh kaedah PPS adalah berdasarkan perwakilan sempadan dan mudah untuk diubah memandangkan ia dicirikan oleh data yang bertabur di sekitar sempadan. Dalam kajian ini. bennik normal SDM dansel-selsabit yang dijanaberdasarkan kepada perwakilan PPS telah dibina mengunakan program MATLAB. Dapatan kajian menunjukkan bahawa kaedah PPS sesuai untuk mewakili bentuk normal SDM dan sel-sel sabit. Selain itu, data berkaitan jejari dan tinggi SDM normal dan salah satu bentuk sel sabit digunakan untuk mendapatkan empat persamaan matematik. Persamaan-persamaan ini boleh digunakan untuk ramalan akan datang dalam mereka bentuk kedua-dua SDM normal dan sel sabit. Kesimpulannya, kaedah PPS boleh menjana perwakilan permukaan parametrik licin bagi sebarang bentuk sel darah. Kajian memberi implikasi bahawa kaedah PPS mampu menjana permukaan geometri yang kompleks.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

Bone marrow produces the red blood cells (RBC) or erythrocytes of the blood (Hasitha, 2014), Hemoglobin is the protein inside red blood cells that conveys oxygen. Red blood cells also get rid of carbon dioxide from human body, moving it to the lungs for breathing procedure. Foods full of iron help us preserve healthy red blood cells. Vitamins are also essential to form healthy red blood cells. Such as vitamin E, which can be found in foods like dark green vegetables, mango, avocados and nuts and seeds; vitamins B2, B12, and B3, found in foods such as whole grains, eggs, and bananas; and folate, available in dried beans, orange juice, lentils, green leafy vegetables and fortified cereals.

The normal RBC's volume in humans is about 80 to 100 femtoliters (fL= 10^{-15} L) (Kleinbongrad, 2006). In metric terms, the size is given in equivalent cubic and the size for RBC is between 6-8 nm (Nanometers Microns). Normally, the range of RBC in Males is: 4.7 to 6.1 million cells per microliter (cells/mcL) and in Females is: 4.2 to 5.4 million cells/mcL. (Klienbongard, 2006).

Anemia is a decrease in the number of red blood cells or hemoglobin level from the normal limit. There are many reasons for anemia. It usually arises because of the weakness and low production of red blood cells and hemoglobin, acute or chronic loss of blood or rapid damage to red blood cells which leads to change red blood cell's shape from spherical shape to form the closest thing to a sickle shape or crescent shape (Charles, 2008).

The red blood cells normally live in a period of 120 days (Klienbongard, 2006). If the disease was hit by the rapid disrepair, the body with the passage of time is unable to compensate for the shortfall arises anemia (Klienbongard, 2006).

For work on problems, theoretical concerning the physical properties of the red blood cell it is imperative to be able to describe the cell shape mathematically. (Vayo, 1982).

1.2 PROBLEM STATEMENT

As it is known, blood is the most important part of our body because it has three main functions which are transport, protection and regulation. Therefore, all mankind need blood in order to let other organs of the body work.

There are many kinds of cells in human blood such as red blood cell (RBC), white blood cell (WBC), platelets and plasma (Christian, 2011).This research focuses on only the red blood cells. The function of RBCs and their importance have been mentioned earlier in this section. We will focus on the shape and size of red blood cells. If a red blood cell is cut into two pieces, what will happen to its image?

There are some mathematical modeling in designing on RBCs such as by differential geometry using Cassinian oval (Borislav and Ivaïlo, 2000) or by formation of the three-dimensional geometry (Hasitha, 2014).

Studying the mathematics of blood could be one of the most interesting fields since it is directly related to our life and health. Recently, there exist a surface generating method that has been used to generate the shape of RBCs, such as polynomial patches that may be not ideally suitable for this work because they would

demand a wide arrangement of control points and weights to represent a shape RBC. Another method is using parametric equation and Jacobian elliptic functions (Vayo, 1982) but it takes a lot of time and uses many equations. Also the method that based on some deformation of the Cassinian oval (Ivailo, Mariana, Peter and Vassil, 2011) this is also is not good enough because it is concerned with the determination of explicit parametric equations of several plane curves.

Another method that can be used to design RBCs is a technique based on the use of partial differential equation (PDE) known as the PDE method. This is the best and easiest method as it depends on the boundary condition and it is possible for every geometry to be generated and controlled by few design parameters (Nur and Jamaludin, 2004), Therefore, this work will focus on the boundary conditions (BC) to generate the red blood cell in a simple way.

1.3 RESEARCH OBJECTIVES

I. The first objective of this work is to model the geometry of a blood cell using the PDE method in order to represent red blood cell. The surfaces are generated as a combination of several fourth order patches.

II. Another objective is to study a shape of the RBCs of those who are diseased with Anemia as it has a direct effect on the shape of the red blood cells. The difference between the shape of a normal RBC and an infected red blood cell will be investigated. The author will study the changes that occur to the shape of RBCs when a person is infected by the disease. In this work the geometry of the infected red blood cells is generated by changing radius, height and center of the normal red blood cell.

1.4 STRUCTURE OF THE THESIS

This chapter begins with a general introduction to our research followed by a problem statement and two objectives. The remaining parts of this report are as follow: Chapter 2 explicates a coordinating presentation of RBC shape. It also includes PDE method and its boundary conditions with some Figures. Chapter 3 discusses the method to sketch the shape of normal red blood cell from several boundary conditions. Chapter 4 sketched two shape of sickle cell and compared one of them with normal RBC mathematically. Chapter 5 contains conclusions and future work.

CHAPTER 2

LITERATURE REVIEW

2.1 EXPLICIT COORDINATE PRESENTATIONS OF RBC SHAPE

The geometrical model of the RBC based on the Cassinian oval is considered. This notable plane curve is defined as a geometrical locus of the points for which the product of the distances from two fixed points G_1 and G_2 is a constant c^2, when the distance (G_1, G_2) between G_1 and G_2 is $2a$. It is given by the equation

$$(X^2 + Z^2 + a^2)^2 - 4a^2X^2 = c^2. \tag{2.1}$$

It is obvious that the above curve is symmetric with respect to both axes and the origin. Actually its shape depends on the precise relationship of the geometrical parameters a and c.

The assumed that $a < c < a\sqrt{2}$ the Bernoullian lemniscate

$$(X^2 + Z^2)^2 = 2a^2(X^2 - Z^2). \tag{2.2}$$

The general meaning of the last notion is that the rectangular coordinates *X,Z* the plane satisfy an algebraic equation

$$G(X,Z)=0, \qquad (2.3)$$

where *G (X,Z)* is a polynomial function in its variables.

In Cartesian coordinates, the RBC surface is described by the equation

$$(a^2 + x^2 + y^2 + z^2)^2 - 4a^2(x^2 + y^2) = c^2. \qquad (2.4)$$

(Borislav and Ivailo, 2000).

2.1.1 POLAR COORDINATE

Let us introduce the standard polar coordinates in the plane

$$x = X\cos(\emptyset), \quad y = X\sin(\emptyset), \quad X \in R^+, \quad \emptyset \in [0, 2\pi] \qquad (2.5)$$

Inserting these coordinates into (2.4) and solving it for z one gets

$$z = \pm\sqrt{\sqrt{c^4 + 4a^2X^2} - a^2 - X^2}. \qquad (2.6)$$

The range interval for X is $[0, \sqrt{c^2 + a^2}]$ and the positive (negative) sign corresponds to that part of the surface which is above (below) the polar plane.

2.1.2 SPHERICAL POLAR COORDINATES

The change of variables in this case is:

$$x = r\sin(\theta)\cos(\emptyset),$$

$$y = r\sin(\theta)\sin(\emptyset),$$

$$z = r\cos(\theta).$$

and after some works the surface equation (2.4) takes the form

$$r^4 + 2a^2r^2\cos(2\theta) + a^4 = c^4. \qquad (2.7)$$

Again it turns out convenient to solve last equation for $z = r\cos(\theta)$ which this time returns

$$z = \pm\frac{\sqrt{c^4 - (a^2 - r^2)^2}}{2a}, \quad r \in \left[\sqrt{c^2 - a^2}, \sqrt{c^2 + a^2}\right] \tag{2.8}$$

with the same meaning of signs, and respectively

$$x = \frac{\sqrt{(a^2 + r^2)^2 - c^4}}{2a}\cos(\emptyset),\ y = \frac{\sqrt{(a^2 + r^2)^2 - c^4}}{2a}\sin(\emptyset). \tag{2.9}$$

Figure 2.1. Half section of RBC drawn using Polar Coordinates via Elliptic Functions (Borislav and Ivailo, 2000)

2.1.3 POLAR COORDINATES VIA ELLIPTIC PDE

Figure 2.1 has been sketched by using diameter and thickness of RBC reveal range from a point say *d* to another point say *g* that are for diameter also for thickness form point but less than *d* to another point less than *g*, which are depending on minimal and maximal values of volume and surface area (Borislav and Ivailo, 2000).

2.2 PDE METHOD

The PDE method was firstly presented into the area of combination shape generation in Computer Aided Design (CAD) by Bloor and Wilson (1989). In recent years, PDE based shapes have widened their uses in shape explanation specially in modeling and graphics, optimization and design analysis (González Castro et al., 2008).

The PDE method produces a parametric surface based on the use of elliptic PDEs, defined by two parameters u and v in a finite region $\Omega \subset R^2$ where $0 \leq v \leq 2\pi$ and $0 \leq u \leq 1$. The general form of an elliptic PDE over a two-dimensional parametric domain is given by

$$\left(\frac{\partial^2}{\partial u^2} + \propto^2 \frac{\partial^2}{\partial v^2}\right)^{2\beta} \underline{X}(u,v) = 0. \tag{2.10}$$

where $\underline{X}(u,v)$ is the function defining a surface in 3D space in a domain Ω while $\propto$ is an essential parameter controlling the relation smoothness of the surface in the u direction and β defines the order of the PDE, and they are limited to $\propto \geq 1$ and $\beta \geq 1$ (Ugail, 2006). Note that, equation (2.10) is biharmonic if $\propto$ and β are set to be 1. The full three-dimensional representation of $\underline{X}(u,v)$ can be written of the form

$$\underline{X}(u,v)=(\underline{X}_x(u,v), \underline{X}_y(u,v), \underline{X}_z(u,v)). \tag{2.11}$$

where it maps a point in (u,v) into a point in 3D space such that $R^2(\Omega) \rightarrow E^3$.

For this work, the approach undertakes is based on the approximate solution of equation (2.10). The PDE method generates a smooth surface patch by solving equation (2.10) subject to a set of periodic boundary conditions that are imposed at the edge of the surface. By taking $\beta = 1$, equation (2.10) transforms to the general form of a fourth-order Elliptic PDE and is solved analytically based on a set of 4 boundary conditions which relate how $\underline{X}(u,v)$ and its normal derivatives, $\frac{\partial \underline{X}}{\partial u}$ vary along $\partial\Omega$.

$$\underline{X}(0,v) = \underline{P}_1(v), \tag{2.12}$$

$$\underline{X}(1,v) = \underline{P}_2(v), \tag{2.13}$$

$$\frac{\partial \underline{X}}{\partial u}(0,v) = \underline{d}_1(v), \tag{2.14}$$

$$\frac{\partial \underline{X}}{\partial u}(1,v) = \underline{d}_2(v). \quad (2.15)$$

PDE with boundary conditions, can be solved by using different methods such as separation of variables, integral transform and finite element method (FE Method). Equations (2.12) and (2.13) which are the first (BC1) and second (BC2) boundary conditions respectively, determine the position at the edges of the surface patch at $u = 0$ and $u = 1$ while equations (2.14) and (2.15) are the third (BC3) and fourth (BC4) boundary conditions respectively define the values of the normal derivatives at the respective boundary of the surface. The derivative conditions play an important role in determining the complete shape of the surface (Ugail, 2006). They are defined by the derivative vector along the boundary curves, where the size and the direction of the derivative vector are determined by the difference between every point on the derivative curve and a related point on the boundary curve. The shape of the surface can be manipulated interactively by changing the direction of the derivative vector and the size.

The solution to equation (2.10) together with the BCs is thus found using the separation of variables method and can be written as:

$$\underline{X}(u,v) = \underline{A}_0 + \sum_{n=1}^{\infty}[\underline{A}_n(u)\cos(nv) + \underline{B}_n(u)\sin(nv)] \quad (2.16)$$

where

$$\underline{A}_0 = \sum_{m=1}^{4} a_{0m}\, u^{m-1}, \quad (2.17)$$

$$\underline{A}_n = (a_{n1} + a_{n3}u)\, e^{\alpha nu} + (a_{n2} + a_{n4}u)\, e^{-\alpha nu}, \quad (2.18)$$

$$\underline{B}_n = (b_{n1} + b_{n3}u)\, e^{\alpha nu} + (b_{n2} + b_{n4}u)\, e^{-\alpha nu}, \quad (2.19)$$

Now equation (2.16) has to be changed with respect to x-axis, y-axis and z-axis, we obtain that

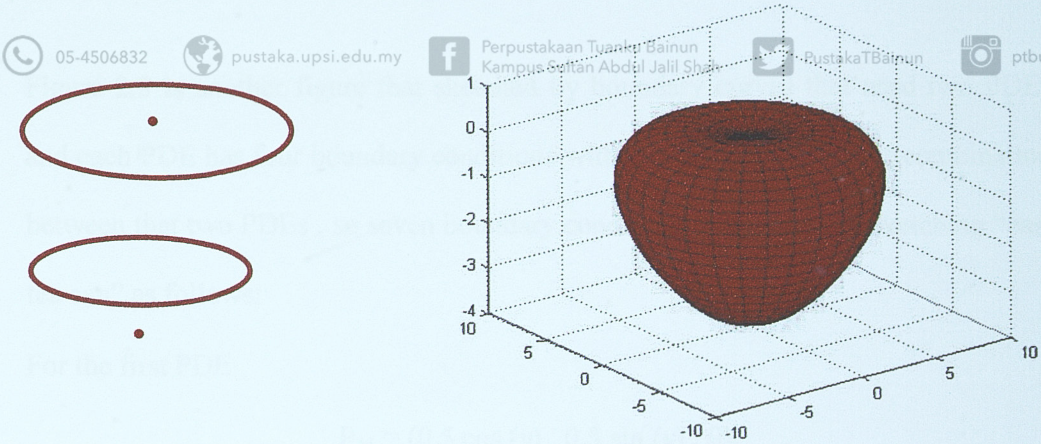
$$\underline{X}_x(u,v) = \underline{A}_{0x} + \sum_{n=1}^{\infty}[\underline{A}_{nx}(u)\cos(nv) + \underline{B}_{nx}(u)\sin(nv)],$$

$$\underline{X}_y(u,v) = \underline{A}_{0y} + \sum_{n=1}^{\infty}[\underline{A}_{ny}(u)\cos(nv) + \underline{B}_{ny}(u)\sin(nv)],$$

$$\underline{X}_z(u,v) = \underline{A}_{0z} + \sum_{n=1}^{\infty}[\underline{A}_{nz}(u)\cos(nv) + \underline{B}_{nz}(u)\sin(nv)].$$

Notice that equation (2.16) has properties of the analytic solution that let us to reproduce the surface shape for every boundary conditions. The term $\underline{A}_0$ in equation (2.16) is a cubic polynomial of the parameter u. It is observed that for every point $\underline{X}(u,v)$ on the surfaces the term $\sum_{n=1}^{\infty}[\underline{A}_n(u)\cos(nv) + \underline{B}_n(u)\sin(nv)]$ in equation (2.16) represent the radial position of the point $\underline{X}(u,v)$ relative to a point $\underline{A}_0$ (Ugail, 2003).

Since $\underline{A}_0$ is a cubic polynomial of the parameter u and lies inside the periodic surface patch so the solution system defined in equation (2.16) a surface point $\underline{X}(u,v)$ perhaps attention as being collected the sum of a vector $\underline{A}_0$ giving the position of the shape of the surface, while the term $\sum_{n=1}^{\infty}[\underline{A}_n(u)\cos(nv) + \underline{B}_n(u)\sin(nv)]$ defining the radius vectors. This section discuss some example of PDE method which are generated by boundary curves.



(a) **(b)**

Figure 2.2. Description of PDE surface. (a) boundary curves (b) surface patch

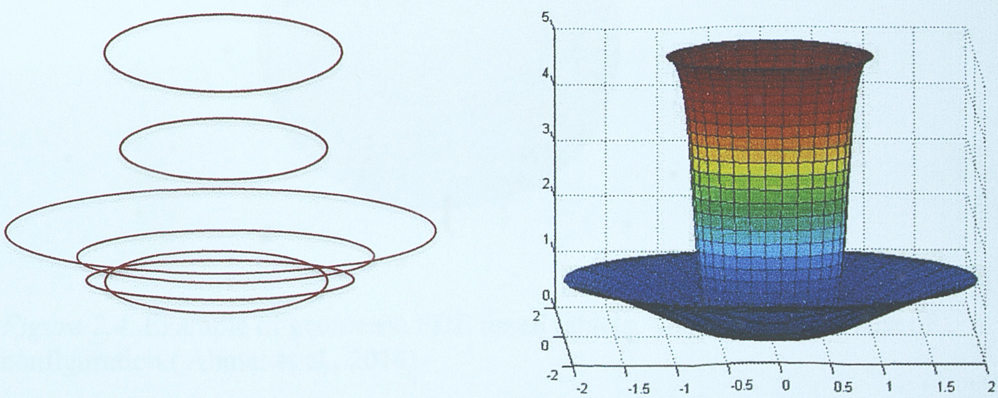
Figure 2.2 shows the surface patch that sketched by one PDE and this PDE used four boundary conditions as follows:

$$P_1 = (0\cos(v), 0\sin(v), -3),$$

$$P_2 = (0\cos(v), 0\sin(v), 0.1),$$

$$d_1 = (2\cos(v), 2\sin(v), -10),$$

$$d_2 = (55\cos(v), 55\sin(v), -5).$$



(a) **(b)**

Figure 2.3. Description of PDE surface. (a) boundary curves (b) surface patch

Figure 2.3 is another figure that sketched by boundary curves that used two PDEs and each PDE has four boundary conditions with one boundary curve is combination between that two PDEs , so seven boundary conditions were used for sketching "east tea can" as follows:

For the first PDE

$$P_{11} = (0.5\cos(v)\,,\,0.5\sin(v)\,,\,0),$$

$$P_{12} = (2\cos(v)\,,\,2\sin(v)\,,\,1),$$

$$d_{11} = (0.7\cos(v)\,,\,0.7\sin(v)\,,\,0.1),$$

$$d_{12} = (1\cos(v)\,,\,1\sin(v)\,,\,0.001).$$

for the second PDE

$$P_{21} = (0.5\cos(v)\,,\,0.5\sin(v)\,,\,0),$$

$$P_{22} = (1\cos(v)\,,\,1\sin(v)\,,\,5),$$

$$d_{21} = (1\cos(v)\,,\,1\sin(v)\,,\,0.1),$$

$$d_{22} = (1\cos(v)\,,\,1\sin(v)\,,\,0.001).$$

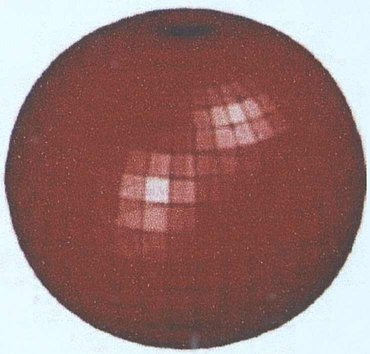


Figure 2.4. Example of geometric PDE based tablets. The resulting surface of configuration.(Ahmat et al., 2014)

Figure 2.4 shows the conditions in terms of the curves that generate a spherical tablet, this shape has sketched by using two PDEs and is symmetric about x-axis so let us consider the boundary curves for PDE one used to generate the upper hemisphere of a sphere with radius 0.6mm. In particular, the coordinates for all boundary conditions are as follows:

$$P_{11} = (0.6 \cos(v), 0.6 \sin(v), 0),$$

$$P_{12} = (0 \cos(v), 0 \sin(v), 0.6),$$

$$d_{11} = (0.4 \cos(v), 0.4 \sin(v), 2),$$

$$d_{12} = (0.001 \cos(v), 0.00\ 1 \sin(v), 0.001).$$

2.3 CONCLUSION

RBC has been sketched by Casinian oval by using diameter and thickness (Borislav and Ivailo, 2000), the PDE method has four boundary conditions, two of these boundary conditions represent the base and height of a PDE-based object, refer to equation (2.12) and (2.13) respectively, while the other two boundary condition which are equation (2.14) and (2.15) define the value of the normal derivative which represent the body of the object. (Ugail, 2006).

CHAPTER 3

DESIGNING NORMAL RED BLOOD CELL

3.1 INTRODUCTION

This chapter offers details on the methodology that has been used in this research. It explains in detail on how a red blood cell is created by utilizing the PDE method.

3.2 PARAMETRIC DESIGN OF A BLOOD CELL USING THE PDE METHOD

This section argues on how to use the PDE method for designing a generic blood cell shape. The geometric model representing an oblate spheroid used during this work has been obtained using 13 generating curves to produce a surface collected of four patches, since every surface patch needs 4 boundary curves. The curves are generated using MATLAB. Every curve is represented by several points from 0 to 2π and the last point confirms that each curve is closed properly. Every curve represents a circle of a given radius at a particular height. In order to use the PDE method to design an oblate spheroid, the designing procedures are divided into several steps. First, the top hemisphere (Patch 1) is generated followed by the concave surface of that hemisphere (Patch 2) is designed as shown in Figure 3.1.

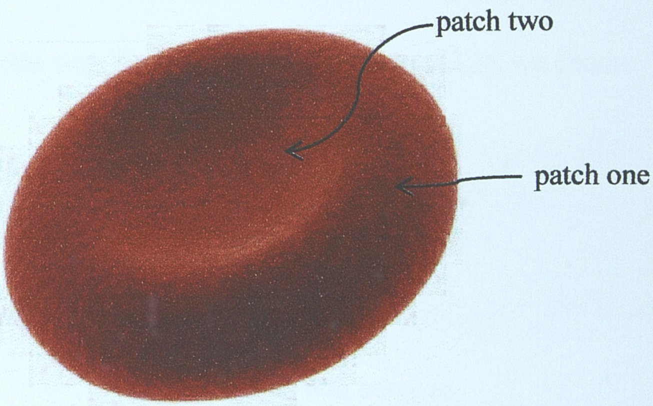


Figure 3.1. Natural Red blood cell (Rachael R. 2010)