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MODELING THE SHAPE OF RED BLOOD CELL USING THE PDE METHOD

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(MATHEMATICS)
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ABSTRACT

This study aims to model the shape of a normal red blood cell (RBC) and two shapes of sickle cells using the partial differential equation (PDE). The development of this technique was based on the use of an elliptic PDE and a set of four periodic boundary conditions. The PDE method can generate surfaces of geometries from a small number of parameters. Furthermore, the shape of the surfaces generated by the PDE method is based on a boundary representation and can easily be modified since it is characterized by data distributed around the boundaries. In this study, the shapes of the generated PDE-based representation of a normal RBC and sickle cell has been sketched by using MATLAB program. The findings showed that the PDE method is suitable for representing the shape of a normal RBC and sickle cells. Besides that, the data regarding the radius and height from the normal RBC and one of the sickle cells, are then used to obtain four equations. These equations can be utilized for future prediction in designing both normal RBC and sickle cells. In conclusion, the PDE method can generate smooth parametric surface representations of any given shape of blood cells. The study implicates that the PDE method is capable of generating surfaces of complex geometries.

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PERMODELAN BENTUK SEL DARAH MERAH MENGGUNAKAN KAEDAH PPS

ABSTRAK

Kajian ini bertujuan untuk membina model bentuk normal sel darah merah (SDM) dan dua bentuk sel sabit menggunakan persamaan pembezaan separa (PPS). Kaedah untuk memperkenalkan teknik ini adalah berasaskan kepada hasil penyelesaian PPS eliptik bersama empat syarat sempadan yang berkala. Kaedah PPS mampu menjana permukaan geometri dengan bilangan parameter yang sedikit. Tambahan pula, permukaan bentuk yang dijana oleh kaedah PPS adalah berdasarkan perwakilan sempadan dan mudah untuk diubah memandangkan ia dicirikan oleh data yang bertabur di sekitar sempadan. Dalam kajian ini. bennik normal SDM dansel-selsabit yang dijanaberdasarkan kepada perwakilan PPS telah dibina mengunakan program MATLAB. Dapatan kajian menunjukkan bahawa kaedah PPS sesuai untuk mewakili bentuk normal SDM dan sel-sel sabit. Selain itu, data berkaitan jejari dan tinggi SDM normal dan salah satu bentuk sel sabit digunakan untuk mendapatkan empat persamaan matematik. Persamaan-persamaan ini boleh digunakan untuk ramalan akan datang dalam mereka bentuk kedua-dua SDM normal dan sel sabit. Kesimpulannya, kaedah PPS boleh menjana perwakilan permukaan parametrik licin bagi sebarang bentuk sel darah. Kajian memberi implikasi bahawa kaedah PPS mampu menjana permukaan geometri yang kompleks.

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CHAPTER 1

INTRODUCTION

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1.1 INTRODUCTION

Bone marrow produces the red blood cells (RBC) or erythrocytes of the blood (Hasitha, 2014), Hemoglobin is the protein inside red blood cells that conveys oxygen. Red blood cells also get rid of carbon dioxide from human body, moving it to the lungs for breathing procedure. Foods full of iron help us preserve healthy red blood cells. Vitamins are also essential to form healthy red blood cells. Such as vitamin E, which can be found in foods like dark green vegetables, mango, avocados and nuts and seeds; vitamins B2, B12, and B3, found in foods such as whole grains, eggs, and bananas; and folate, available in dried beans, orange juice, lentils, green leafy vegetables and fortified cereals.

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The normal RBC's volume in humans is about 80 to 100 femtoliters (fL= 10^{-15} L) (Kleinbongrad, 2006). In metric terms, the size is given in equivalent cubic and the size for RBC is between 6-8 nm (Nanometers Microns). Normally, the range of RBC in Males is: 4.7 to 6.1 million cells per microliter (cells/mcL) and in Females is: 4.2 to 5.4 million cells/mcL. (Klienbongard, 2006).

Anemia is a decrease in the number of red blood cells or hemoglobin level from the normal limit. There are many reasons for anemia. It usually arises because of the weakness and low production of red blood cells and hemoglobin, acute or chronic loss of blood or rapid damage to red blood cells which leads to change red blood cell's shape from spherical shape to form the closest thing to a sickle shape or crescent shape (Charles, 2008).

The red blood cells normally live in a period of 120 days (Klienbongard, 2006). If the disease was hit by the rapid disrepair, the body with the passage of time is unable to compensate for the shortfall arises anemia (Klienbongard, 2006).

For work on problems, theoretical concerning the physical properties of the red blood cell it is imperative to be able to describe the cell shape mathematically. (Vayo, 1982).

1.2 PROBLEM STATEMENT

As it is known, blood is the most important part of our body because it has three main functions which are transport, protection and regulation. Therefore, all mankind need blood in order to let other organs of the body work.

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There are many kinds of cells in human blood such as red blood cell (RBC), white blood cell (WBC), platelets and plasma (Christian, 2011).This research focuses on only the red blood cells. The function of RBCs and their importance have been mentioned earlier in this section. We will focus on the shape and size of red blood cells. If a red blood cell is cut into two pieces, what will happen to its image?

There are some mathematical modeling in designing on RBCs such as by differential geometry using Cassinian oval (Borislav and Ivailo, 2000) or by formation of the three-dimensional geometry (Hasitha, 2014).

Studying the mathematics of blood could be one of the most interesting fields since it is directly related to our life and health. Recently, there exist a surface generating method that has been used to generate the shape of RBCs, such as polynomial patches that may be not ideally suitable for this work because they would demand a wide arrangement of control points and weights to represent a shape RBC. Another method is using parametric equation and Jacobian elliptic functions (Vayo, 1982) but it takes a lot of time and uses many equations. Also the method that based on some deformation of the Cassinian oval (Ivailo, Mariana, Peter and Vassil, 2011) this is also is not good enough because it is concerned with the determination of explicit parametric equations of several plane curves.

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Another method that can be used to design RBCs is a technique based on the use of partial differential equation (PDE) known as the PDE method. This is the best and easiest method as it depends on the boundary condition and it is possible for every geometry to be generated and controlled by few design parameters (Nur and Jamaludin, 2004), Therefore, this work will focus on the boundary conditions (BC) to generate the red blood cell in a simple way.

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1.3 RESEARCH OBJECTIVES

I. The first objective of this work is to model the geometry of a blood cell using the PDE method in order to represent red blood cell. The surfaces are generated as a combination of several fourth order patches.

II. Another objective is to study a shape of the RBCs of those who are diseased with Anemia as it has a direct effect on the shape of the red blood cells. The difference between the shape of a normal RBC and an infected red blood cell will be investigated. The author will study the changes that occur to the shape of RBCs when a person is infected by the disease. In this work the geometry of the infected red blood cells is generated by changing radius, height and center of the normal red blood cell.

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1.4 STRUCTURE OF THE THESIS

This chapter begins with a general introduction to our research followed by a problem statement and two objectives. The remaining parts of this report are as follow: Chapter 2 explicates a coordinating presentation of RBC shape. It also includes PDE method and its boundary conditions with some Figures. Chapter 3 discusses the method to sketch the shape of normal red blood cell from several boundary conditions. Chapter 4 sketched two shape of sickle cell and compared one of them with normal RBC mathematically. Chapter 5 contains conclusions and future work.

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CHAPTER 2

LITERATURE REVIEW

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2.1 EXPLICIT COORDINATE PRESENTATIONS OF RBC SHAPE

The geometrical model of the RBC based on the Cassinian oval is considered. This notable plane curve is defined as a geometrical locus of the points for which the product of the distances from two fixed points G_1 and G_2 is a constant c^2, when the distance (G_1, G_2) between G_1 and G_2 is $2a$. It is given by the equation

$$(X^2 + Z^2 + a^2)^2 - 4a^2X^2 = c^2. \quad (2.1)$$

It is obvious that the above curve is symmetric with respect to both axes and the origin. Actually its shape depends on the precise relationship of the geometrical parameters a and c.

The assumed that $a < c < a\sqrt{2}$ the Bernoullian lemniscate

$$(X^2 + Z^2)^2 = 2a^2(X^2 - Z^2). \quad (2.2)$$

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The general meaning of the last notion is that the rectangular coordinates X,Z the plane satisfy an algebraic equation

$$G(X,Z)=0, \tag{2.3}$$

where $G(X,Z)$ is a polynomial function in its variables.

In Cartesian coordinates, the RBC surface is described by the equation

$$(a^2 + x^2 + y^2 + z^2)^2 - 4a^2(x^2 + y^2) = c^2. \tag{2.4}$$

(Borislav and Ivailo, 2000).

2.1.1 POLAR COORDINATE

Let us introduce the standard polar coordinates in the plane

$$x = X\cos(\emptyset), \quad y = X\sin(\emptyset), \quad X \in R^+, \quad \emptyset \in [0, 2\pi] \tag{2.5}$$

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Inserting these coordinates into (2.4) and solving it for z one gets

$$z = \pm\sqrt{\sqrt{c^4 + 4a^2X^2} - a^2 - X^2}. \tag{2.6}$$

The range interval for X is $[0, \sqrt{c^2 + a^2}]$ and the positive (negative) sign corresponds to that part of the surface which is above (below) the polar plane.

2.1.2 SPHERICAL POLAR COORDINATES

The change of variables in this case is:

$$x = r\sin(\theta)\cos(\emptyset),$$

$$y = r\sin(\theta)\sin(\emptyset),$$

$$z = r\cos(\theta).$$

and after some works the surface equation (2.4) takes the form

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$$r^4 + 2a^2r^2\cos(2\theta) + a^4 = c^4. \tag{2.7}$$

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Again it turns out convenient to solve last equation for $z = r\,cos\,(\theta)$ which this time returns

$$z = \pm \frac{\sqrt{c^4 - (a^2 - r^2)^2}}{2\,a}, \quad r \in \left[\sqrt{c^2 - a^2}, \sqrt{c^2 + a^2}\right] \tag{2.8}$$

with the same meaning of signs, and respectively

$$x = \frac{\sqrt{(a^2+r^2)^2 - c^4}}{2a}\cos(\emptyset), \; y = \frac{\sqrt{(a^2+r^2)^2 - c^4}}{2a}\sin(\emptyset). \tag{2.9}$$



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Figure 2.1. Half section of RBC drawn using Polar Coordinates via Elliptic Functions (Borislav and Ivailo, 2000)

2.1.3 POLAR COORDINATES VIA ELLIPTIC PDE

Figure 2.1 has been sketched by using diameter and thickness of RBC reveal range from a point say *d* to another point say *g* that are for diameter also for thickness form point but less than *d* to another point less than *g*, which are depending on minimal and maximal values of volume and surface area (Borislav and Ivailo, 2000).

2.2 PDE METHOD

The PDE method was firstly presented into the area of combination shape generation in Computer Aided Design (CAD) by Bloor and Wilson (1989). In recent years, PDE

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based shapes have widened their uses in shape explanation specially in modeling and graphics, optimization and design analysis (González Castro et al., 2008).

The PDE method produces a parametric surface based on the use of elliptic PDEs, defined by two parameters u and v in a finite region $\Omega \subset R^2$ where $0 \leq v \leq 2\pi$ and $0 \leq u \leq 1$. The general form of an elliptic PDE over a two-dimensional parametric domain is given by

$$\left(\frac{\partial^2}{\partial u^2} + \propto^2 \frac{\partial^2}{\partial v^2}\right)^{2\beta} \underline{X}(u,v) = 0. \tag{2.10}$$

where $\underline{X}(u,v)$ is the function defining a surface in 3D space in a domain Ω while $\propto$ is an essential parameter controlling the relation smoothness of the surface in the u direction and β defines the order of the PDE, and they are limited to $\propto \geq 1$ and $\beta \geq 1$ (Ugail, 2006). Note that, equation (2.10) is biharmonic if $\propto$ and β are set to be 1. The full three-dimensional representation of $\underline{X}(u,v)$ can be written of the form

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$$\underline{X}(u,v)=(\underline{X}_x(u,v), \underline{X}_y(u,v), \underline{X}_z(u,v)). \tag{2.11}$$

where it maps a point in (u,v) into a point in 3D space such that $R^2(\Omega) \rightarrow E^3$.

For this work, the approach undertakes is based on the approximate solution of equation (2.10). The PDE method generates a smooth surface patch by solving equation (2.10) subject to a set of periodic boundary conditions that are imposed at the edge of the surface. By taking $\beta = 1$, equation (2.10) transforms to the general form of a fourth-order Elliptic PDE and is solved analytically based on a set of 4 boundary conditions which relate how $\underline{X}(u,v)$ and its normal derivatives, $\frac{\partial \underline{X}}{\partial u}$ vary along $\partial\Omega$.

$$\underline{X}(0,v) = \underline{P}_1(v), \tag{2.12}$$

$$\underline{X}(1,v) = \underline{P}_2(v), \tag{2.13}$$

$$\frac{\partial \underline{X}}{\partial u}(0,v) = \underline{d}_1(v), \tag{2.14}$$

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$$\frac{\partial \underline{X}}{\partial u}(1,v) = \underline{d}_2(v). \tag{2.15}$$

PDE with boundary conditions, can be solved by using different methods such as separation of variables, integral transform and finite element method (FE Method). Equations (2.12) and (2.13) which are the first (BC1) and second (BC2) boundary conditions respectively, determine the position at the edges of the surface patch at $u = 0$ and $u = 1$ while equations (2.14) and (2.15) are the third (BC3) and fourth (BC4) boundary conditions respectively define the values of the normal derivatives at the respective boundary of the surface. The derivative conditions play an important role in determining the complete shape of the surface (Ugail, 2006). They are defined by the derivative vector along the boundary curves, where the size and the direction of the derivative vector are determined by the difference between every point on the derivative curve and a related point on the boundary curve. The shape of the surface can be manipulated interactively by changing the direction of the derivative vector and the size.

The solution to equation (2.10) together with the BCs is thus found using the separation of variables method and can be written as:

$$\underline{X}(u,v) = \underline{A}_0 + \sum_{n=1}^{\infty} [\underline{A}_n(u)\cos(nv) + \underline{B}_n(u)\sin(nv)] \tag{2.16}$$

where

$$\underline{A}_0 = \sum_{m=1}^{4} a_{0m}\, u^{m-1}, \tag{2.17}$$

$$\underline{A}_n = (a_{n1} + a_{n3}u)\, e^{\propto nu} + (a_{n2} + a_{n4}u)\, e^{-\propto nu}, \tag{2.18}$$

$$\underline{B}_n = (b_{n1} + b_{n3}u)\, e^{\propto nu} + (b_{n2} + b_{n4}u)\, e^{-\propto nu}, \tag{2.19}$$

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Now equation (2.16) has to be changed with respect to x-axis, y-axis and z-axis, we obtain that

$$\underline{X}_x(u,v) = \underline{A}_{0x} + \sum_{n=1}^{\infty}\left[\underline{A}_{nx}(u)\cos(nv) + \underline{B}_{nx}(u)\sin(nv)\right],$$

$$\underline{X}_y(u,v) = \underline{A}_{0y} + \sum_{n=1}^{\infty}\left[\underline{A}_{ny}(u)\cos(nv) + \underline{B}_{ny}(u)\sin(nv)\right],$$

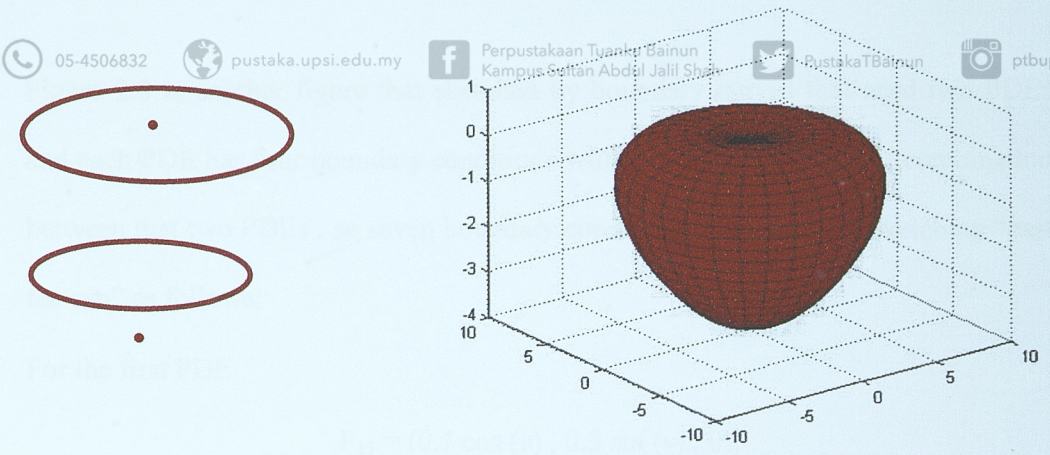
$$\underline{X}_z(u,v) = \underline{A}_{0z} + \sum_{n=1}^{\infty}\left[\underline{A}_{nz}(u)\cos(nv) + \underline{B}_{nz}(u)\sin(nv)\right].$$

Notice that equation (2.16) has properties of the analytic solution that let us to reproduce the surface shape for every boundary conditions. The term $\underline{A}_0$ in equation (2.16) is a cubic polynomial of the parameter u. It is observed that for every point $\underline{X}(u,v)$ on the surfaces the term $\sum_{n=1}^{\infty}[\underline{A}_n(u)\cos(nv) + \underline{B}_n(u)\sin(nv)]$ in equation (2.16) represent the radial position of the point $\underline{X}(u,v)$ relative to a point $\underline{A}_0$ (Ugail, 2003).

Since $\underline{A}_0$ is a cubic polynomial of the parameter u and lies inside the periodic surface patch so the solution system defined in equation (2.16) a surface point $\underline{X}(u,v)$ perhaps attention as being collected the sum of a vector $\underline{A}_0$ giving the position of the shape of the surface, while the term $\sum_{n=1}^{\infty}[\underline{A}_n(u)\cos(nv) + \underline{B}_n(u)\sin(nv)]$ defining the radius vectors. This section discuss some example of PDE method which are generated by boundary curves.

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(a) **(b)**

Figure 2.2. Description of PDE surface. (a) boundary curves (b) surface patch

Figure 2.2 shows the surface patch that sketched by one PDE and this PDE used four boundary conditions as follows:

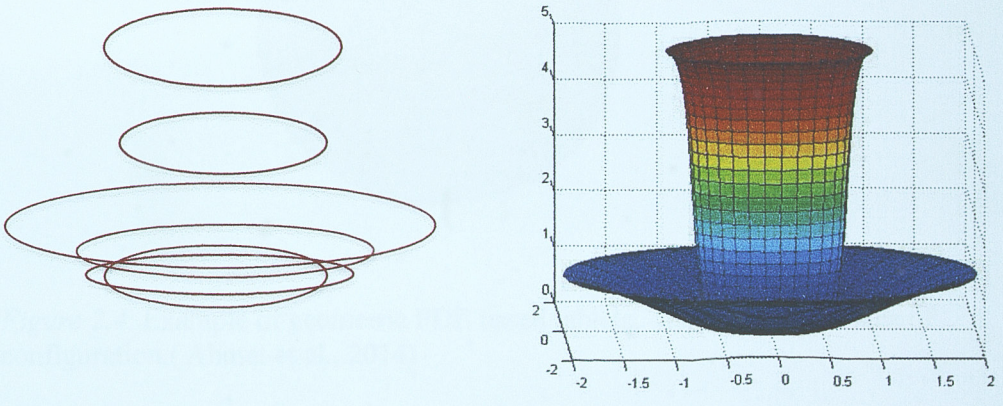
$$P_1 = (0 \cos(v), 0 \sin(v), -3),$$

$$P_2 = (0 \cos(v), 0 \sin(v), 0.1),$$

$$d_1 = (2 \cos(v), 2 \sin(v), -10),$$

$$d_2 = (55 \cos(v), 55 \sin(v), -5).$$

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(a) **(b)**

Figure 2.3. Description of PDE surface. (a) boundary curves (b) surface patch

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Figure 2.3 is another figure that sketched by boundary curves that used two PDEs and each PDE has four boundary conditions with one boundary curve is combination between that two PDEs , so seven boundary conditions were used for sketching "east tea can" as follows:

For the first PDE

$$P_{11} = (0.5 \cos(v) , 0.5 \sin(v) , 0),$$

$$P_{12} = (2 \cos(v) , 2 \sin(v) , 1),$$

$$d_{11} = (0.7 \cos(v) , 0.7 \sin(v) , 0.1),$$

$$d_{12} = (1 \cos(v) , 1 \sin(v) , 0.001).$$

for the second PDE

$$P_{21} = (0.5 \cos(v) , 0.5 \sin(v) , 0),$$

$$P_{22} = (1 \cos(v) , 1 \sin(v) , 5),$$

$$d_{21} = (1 \cos(v) , 1 \sin(v) , 0.1),$$

$$d_{22} = (1 \cos(v) , 1 \sin(v) , 0.001).$$

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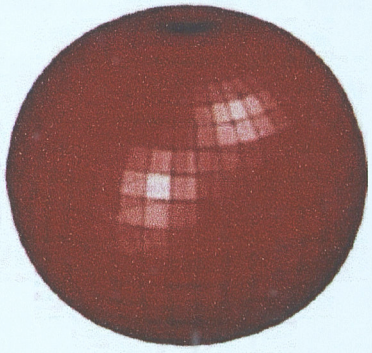


Figure 2.4. Example of geometric PDE based tablets. The resulting surface of configuration.(Ahmat et al., 2014)

Figure 2.4 shows the conditions in terms of the curves that generate a spherical tablet, this shape has sketched by using two PDEs and is symmetric about x-axis so let us

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consider the boundary curves for PDE one used to generate the upper hemisphere of a sphere with radius 0.6mm. In particular, the coordinates for all boundary conditions are as follows:

$$P_{11} = (0.6 \cos(v) , 0.6 \sin(v) , 0),$$

$$P_{12} = (0 \cos(v) , 0 \sin(v) , 0.6),$$

$$d_{11} = (0.4 \cos(v) , 0.4 \sin(v) , 2),$$

$$d_{12} = (0.001 \cos(v) , 0.00\ 1 \sin(v) , 0.001).$$

2.3 CONCLUSION

RBC has been sketched by Casinian oval by using diameter and thickness (Borislav and Ivailo, 2000), the PDE method has four boundary conditions, two of these boundary conditions represent the base and height of a PDE-based object, refer to equation (2.12) and (2.13) respectively, while the other two boundary condition which are equation (2.14) and (2.15) define the value of the normal derivative which represent the body of the object. (Ugail, 2006).

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CHAPTER 3

DESIGNING NORMAL RED BLOOD CELL

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3.1 INTRODUCTION

This chapter offers details on the methodology that has been used in this research. It explains in detail on how a red blood cell is created by utilizing the PDE method.

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3.2 PARAMETRIC DESIGN OF A BLOOD CELL USING THE PDE METHOD

This section argues on how to use the PDE method for designing a generic blood cell shape. The geometric model representing an oblate spheroid used during this work has been obtained using 13 generating curves to produce a surface collected of four patches, since every surface patch needs 4 boundary curves. The curves are generated using MATLAB. Every curve is represented by several points from 0 to 2π and the last point confirms that each curve is closed properly. Every curve represents a circle of a given radius at a particular height. In order to use the PDE method to design an oblate spheroid, the designing procedures are divided into several steps. First, the top hemisphere (Patch 1) is generated followed by the concave surface of that hemisphere (Patch 2) is designed as shown in Figure 3.1.

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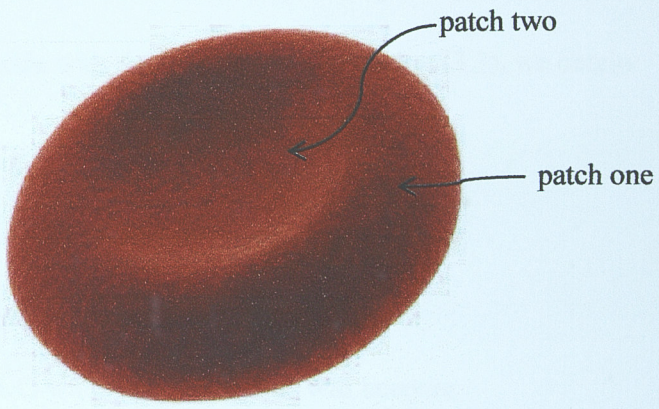


Figure 3.1. Natural Red blood cell (Rachael R. 2010)

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Now we continue the process for designing the bottom hemisphere, by using the same way of designing the top hemisphere. The bottom hemisphere (Patch 3) is generated followed by the concave surface of that hemisphere (Patch 4) is designed.

Combining the top and the bottom hemisphere to get the shape of red blood cell.

As mentioned in chapter two the PDE surface is generated by solving equation (2.16) for n =1, obtained when equations (2.17), (2.18) and (2.19) are substituted into the equation (2.16). Hence the equation (2.16) becomes

$$\underline{X}(u,v)=[\underline{a}_{01}+\underline{a}_{02}\,u+\underline{a}_{03}\,u^2+\underline{a}_{04}\,u^3]+[(\underline{a}_{11}+\underline{a}_{13}\,u)\,e^{\propto u}+(\underline{a}_{12}+\underline{a}_{14}\,u)\,e^{-\propto u}]\,\mathrm{Cos(v)} +[(\underline{b}_{11}+\underline{b}_{13}\,u)\,e^{\propto u}+(\underline{b}_{12}+\underline{b}_{14}\,u)\,e^{-\propto u}]\,\mathrm{sin(v)}. \quad (3.1)$$

The first boundary condition (BC1) when $u = 0$ can be written as

$$\underline{X}(0,v) = (r_1\,cos(v),\ r_1\,sin(v),\ h_1)\,, \quad (3.2)$$

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Depends on equation (2.11) we can change equation (3.2) as:

$$(\underline{X}_x(0,v),\ \underline{X}_y(0,v),\ \underline{X}_z(0,v)) = (r_1\,cos(v),\ r_1\,sin(v),\ h_1). \quad (3.3)$$

where r_1 is the radius of circles in BC1 and h_1 is the height of circles in BC1

By comparing left hand side with right hand side of equation (3.3), we obtain

$$\underline{X}_x(0,v) = r_1\,cos(v), \quad (3.4)$$

$$\underline{X}_y(0,v) = r_1\,sin(v), \quad (3.5)$$

$$\underline{X}_z(0,v) = h_1\,. \quad (3.6)$$

Since u=0 so equation (3.1) with respect to x-axis will become

$$\underline{X}_x(0,v)=[\underline{a}_{01x}]+[\underline{a}_{11x}+\underline{a}_{12x}]\,\mathrm{cos(v)}\ +[\,\underline{b}_{11x}+\underline{b}_{12x}\,]\,\mathrm{sin(v)}.$$

Substitute the above equation into equation (3.4)

$$\underline{a}_{01x}+(\underline{a}_{11x}+\underline{a}_{12x})\,cos(v)+(\underline{b}_{11x}+\underline{b}_{12x})\,sin(v)=r_1\,cos(v),$$

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now by coefficient comparison between two side of above equation we will get

$$\underline{a}_{01x} = 0,$$

$$\underline{a}_{11x} + \underline{a}_{12x} = r_1,$$

$$\underline{b}_{11x} + \underline{b}_{12x} = 0.$$

Equation (3.1) with respect to y-axis after substituting u=0 become

$$\underline{X}_y(0,v)=[\underline{a}_{01y}]+[\underline{a}_{11y} + \underline{a}_{12y}] \cos(v) + [\underline{b}_{11y} + \underline{b}_{12y}] \sin(v).$$

Substitute equation (3.1) with respect to y-axis after making u=0 into equation (3.5)

$$\underline{a}_{01y} + (\underline{a}_{11y} + \underline{a}_{12y}) \cos(v) + (\underline{b}_{11y} + \underline{b}_{12y}) \sin(v) = r_1 \sin(v),$$

Also by coefficient comparison between two side of above equation we will get

$$\underline{a}_{01y} = 0,$$

$$\underline{a}_{11y} + \underline{a}_{12y} = 0,$$

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$$\underline{b}_{11y} + \underline{b}_{12y} = r_1.$$

Again u=0 so equation (3.1) with respect to z-axis will become

$$\underline{X}_z(0,v)=[\underline{a}_{01z}]+[\underline{a}_{11x} + \underline{a}_{12z}] \cos(v) + [\underline{b}_{11z} + \underline{b}_{12z}] \sin(v).$$

for compare above equation with equation (3.6)

$$\underline{a}_{01z} + (\underline{a}_{11z} + \underline{a}_{12z}) \cos(v) + (\underline{b}_{11z} + \underline{b}_{12z}) \sin(v) = h_1,$$

also by coefficient comparison

$$\underline{a}_{01z} = h_1,$$

$$\underline{a}_{11z} + \underline{a}_{12z} = 0,$$

$$\underline{b}_{11z} + \underline{b}_{12z} = 0.$$

By the same way of solving the BC1 shown in equation (2.12), we will solve BC2 which is equation (2.13) when $u = 1$.

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The second BC when u=1 can be written as

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$$(\underline{X}_x(1,v), \underline{X}_y(1,v), \underline{X}_z(1,v)) = (r_2\, cos(v),\ r_2\, sin(v),\ h_2). \tag{3.7}$$

where r_2 and h_2 are the radius and height for the circles in BC2 respectively.

Comparison both sides of equation (3.7).

$$\underline{X}_x(1,v) = r_2\, cos(v),$$

$$\underline{X}_y(1,v) = r_2\, sin(v),$$

$$\underline{X}_z(1,v) = h_2.$$

Now equation (3.1) after substituting u=1 become

$$\underline{X}(1,v)=[\underline{a}_{01}+\underline{a}_{02}+\underline{a}_{03}+\underline{a}_{04}]+[(\underline{a}_{11}+\underline{a}_{13})\,e^{\propto}+(\underline{a}_{12}+\underline{a}_{14})\,e^{-\propto}]\cos(v)+$$

$$[(\underline{b}_{11}+\underline{b}_{13})\,e^{\propto}+(\underline{b}_{12}+\underline{b}_{14})\,e^{-\propto}]\sin(v).$$

Change above equation to the full three-dimension with respect to x-axis, y-axis and z-axis and doing comparison as follows:

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With respect to x-axis: $\underline{X}_x(1,v) = r_2\, cos(v),$

$$[\underline{a}_{01x}+\underline{a}_{02x}+\underline{a}_{03x}+\underline{a}_{04x}]+[(\underline{a}_{11x}+\underline{a}_{13x})\,e^{\propto}+(\underline{a}_{12x}+\underline{a}_{14x})\,e^{-\propto}]\,cos\,(v)+$$

$$[(\underline{b}_{11x}+\underline{b}_{13x})\,e^{\propto}+(\underline{b}_{12x}+\underline{b}_{14x})\,e^{-\propto}]\,sin(v) = r_2\ cos(v).$$

Compare the coefficient between two sides

$$\underline{a}_{01x}+\underline{a}_{02x}+\underline{a}_{03x}+\underline{a}_{04x} = 0,$$

$$(\underline{a}_{11x}+\underline{a}_{13x})\,e^{\propto}+(\underline{a}_{12x}+\underline{a}_{14x})\,e^{-\propto} = r_2,$$

$$(\underline{b}_{11x}+\underline{b}_{13x})\,e^{\propto}+(\underline{b}_{12x}+\underline{b}_{14x})\,e^{-\propto} = 0.$$

With respect to y-axis: $\underline{X}_y(1,v) = r_2\, sin(v)$,

$$[\underline{a}_{01y}+\underline{a}_{02y}+\underline{a}_{03y}+\underline{a}_{04y}]+[(\underline{a}_{11y}+\underline{a}_{13y})\,e^{\propto}+(\underline{a}_{12y}+\underline{a}_{14y})\,e^{-\propto}]\ cos\,(v)+$$

$$[(\underline{b}_{11y}+\underline{b}_{13y})\,e^{\propto}+(\underline{b}_{12y}+\underline{b}_{14y})\,e^{-\propto}]\,sin(v) = r_2\,sin(v).$$

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By comparison coefficient between two sides we obtain

$$\underline{a}_{01y} + \underline{a}_{02y} + \underline{a}_{03y} + \underline{a}_{04y} = 0,$$

$$(\underline{a}_{11y} + \underline{a}_{13y})\, e^{\propto} + (\underline{a}_{12y} + \underline{a}_{14y})\, e^{-\propto} = 0,$$

$$(\underline{b}_{11y} + \underline{b}_{13y})\, e^{\propto} + (\underline{b}_{12y} + \underline{b}_{14y})\, e^{-\propto} = r_2.$$

With respect to z-axis: $\underline{X}_z(1,v) = h_2,$

$$[\underline{a}_{01z} + \underline{a}_{02z} + \underline{a}_{03z} + \underline{a}_{04z}] + [(\underline{a}_{11z} + \underline{a}_{13z})\, e^{\propto} + (\underline{a}_{12z} + \underline{a}_{14z})\, e^{-\propto}]\, cos\,(v) +$$

$$[(\underline{b}_{11z} + \underline{b}_{13z})\, e^{\propto} + (\underline{b}_{12z} + \underline{b}_{14z})\, e^{-\propto}]\, sin(v) = h_2.$$

Again compare coefficients

$$\underline{a}_{01z} + \underline{a}_{02z} + \underline{a}_{03z} + \underline{a}_{04z} = h_2,$$

$$(\underline{a}_{11z} + \underline{a}_{13z})\, e^{\propto} + (\underline{a}_{12z} + \underline{a}_{14z})\, e^{-\propto} = 0,$$

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$$(\underline{b}_{11z} + \underline{b}_{13z})\, e^{\propto} + (\underline{b}_{12z} + \underline{b}_{14z})\, e^{-\propto} = 0.$$

Now let's consider BC3 and BC4 which are shown in equations (2.14) and (2.15) respectively. Firstly, we need to differentiate equation (3.1) with respect to u and then apply both conditions, we eventually obtain

$$\frac{\partial \underline{X}}{\partial u}(u,v) = [\underline{a}_{02} + 2\,\underline{a}_{03}\,u + 3\,\underline{a}_{04}\,u^2] +$$

$$[\propto \underline{a}_{11}\, e^{\propto u} + \propto \underline{a}_{13}\, u\, e^{\propto u} + \underline{a}_{13}\, e^{\propto u} - \propto \underline{a}_{12}\, e^{-\propto u} - \propto \underline{a}_{14}\, u\, e^{-\propto u} + \underline{a}_{14}\, e^{-\propto u}]\, cos\,(v) +$$
$$[\propto \underline{b}_{11}\, e^{\propto u} + \propto \underline{b}_{13}\, u\, e^{\propto u} + \underline{b}_{13}\, e^{\propto u} - \propto \underline{b}_{12}\, e^{-\propto u} - \propto \underline{b}_{14}\, u\, e^{-\propto u} + \underline{b}_{14}\, e^{-\propto u}]\, sin(v). \quad (3.8)$$

When $u = 0$ so equation (3.8) becomes

$$\frac{\partial \underline{X}}{\partial u}(0,v) = [\underline{a}_{02}] + [\propto \underline{a}_{11} + \underline{a}_{13} - \propto \underline{a}_{12} + \underline{a}_{14}\, e^{-\propto u}]\, cos\,(v) +$$

$$[\propto \underline{b}_{11} + \underline{b}_{13} - \propto \underline{b}_{12} + \underline{b}_{14}\, e^{-\propto u}]\, sin(v). \quad (3.9)$$

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The third boundary condition (BC3) when $u = 0$ can be written as

$$(\frac{\partial \underline{X}_x}{\partial u}(0,v), \frac{\partial \underline{X}_y}{\partial u}(0,v), \frac{\partial \underline{X}_z}{\partial u}(0,v)) = (r_3 cos\ (v),\ r_3\ sin(v),\ h_3). \quad (3.10)$$

Note that r_3 and h_3 are the radius and height for the circles in BC3 respectively.

Changing equation (3.9) to the three-dimension with respect to x-axis, y-axis and z-axis and then compare it to the equation (3.10) as follows:

With respect to x-axis $$\frac{\partial \underline{X}_x}{\partial u}(0,v) = r_3\ cos(v).$$

Substitute above equation in equation (3.9)

$$[\underline{a}_{02}] + [\propto \underline{a}_{11} + \underline{a}_{13} - \propto \underline{a}_{12} + \underline{a}_{14} e^{-\propto u}]\ cos\ (v) + [\propto \underline{b}_{11} + \underline{b}_{13} - \propto \underline{b}_{12} + \underline{b}_{14} e^{-\propto u}]\ sin(v) = r_3\ cos(v)$$

Therefore by comparison coefficient between two sides

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$$\underline{a}_{02x} = 0,$$

$$\propto \underline{a}_{11x} + \underline{a}_{13x} - \propto \underline{a}_{12x} + \underline{a}_{14x} = r_3,$$

$$\propto \underline{b}_{11} + \underline{b}_{13} - \propto \underline{b}_{12} + + \underline{b}_{14} = 0.$$

With respect to y-axis $$\frac{\partial \underline{X}_y}{\partial u}(0,v) = r_3\ sin(v).$$

Also substitute above equation in equation (3.9) after then make coefficient comparison

$$\underline{a}_{02y} = 0,$$

$$\propto \underline{a}_{11y} + \underline{a}_{13y} - \propto \underline{a}_{12y} + \underline{a}_{14y} = 0,$$

$$\propto \underline{b}_{11y} + \underline{b}_{13y} - \propto \underline{b}_{12y} + \underline{b}_{14y} = r_3.$$

With respect to z-axis $$\frac{\partial \underline{X}_z}{\partial u}(0,v) = h_3.$$

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Also substitute above equation in equation (3.9) after then make coefficient comparison

$$\underline{a}_{02z} = h_3,$$

$$\propto \underline{a}_{11z} + \underline{a}_{13z} - \propto \underline{a}_{12z} + \underline{a}_{14z} = 0,$$

$$\propto \underline{b}_{11z} + \underline{b}_{13z} - \propto \underline{b}_{12z} + \underline{b}_{14z} = 0.$$

For the BC4, first we have to take $u = 1$, into equation (3.8) we get

$$\frac{\partial \underline{X}}{\partial u}(1,v) = [\underline{a}_{02} + 2\,\underline{a}_{03} + 3\,\underline{a}_{04}] +$$

$$[\propto \underline{a}_{11}\, e^{\propto} + \propto \underline{a}_{13}\, e^{\propto} + \underline{a}_{13}\, e^{\propto} - \propto \underline{a}_{12}\, e^{-\propto} - \propto \underline{a}_{14}\, e^{-\propto} + \underline{a}_{14}\, e^{-\propto}]\, cos\,(v) +$$

$$[\propto \underline{b}_{11}\, e^{\propto} + \propto \underline{b}_{13}\, e^{\propto} + \underline{b}_{13}\, e^{\propto} - \propto \underline{b}_{12}\, e^{-\propto} - \propto \underline{b}_{14}\, e^{-\propto} + \underline{b}_{14}\, e^{-\propto}]\; sin(v).$$

The fourth boundary condition (BC4) when $u = 1$ can be written as

$$\left(\frac{\partial \underline{X}_x}{\partial u}(1,v), \frac{\partial \underline{X}_y}{\partial u}(1,v), \frac{\partial \underline{X}_z}{\partial u}(1,v)\right) = (r_4\, cos(v), r_4\, sin(v), h_4). \qquad (3.11)$$

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where r_4 and h_4 are the radius and height for the circles in BC4 respectively.

Changing equation (3.8) after substituting u=1 to the three-dimension with respect to x-axis, y-axis and z-axis and then compare it to the equation (3.11) as follows:

With respect to x-axis $$\frac{\partial \underline{X}_x}{\partial u}(1,v) = r_4\, cos(v),$$

Implies that

$$[\underline{a}_{02x} + 2\,\underline{a}_{03x} + 3\,\underline{a}_{04x}] + [\propto \underline{a}_{11x}\, e^{\propto} + (1+\propto)\,\underline{a}_{13x}\, e^{\propto} - \propto \underline{a}_{12x}\, e^{-\propto} + (1 - \propto)\,\underline{a}_{14x}\, e^{-\propto}]$$
$$\cos(v)+[\propto \underline{b}_{11x}\, e^{\propto} + (1+\propto)\,\underline{b}_{13x}\, e^{\propto} - \propto \underline{b}_{12x}\, e^{-\propto} + (1 - \propto)\,\underline{b}_{14x}\, e^{-\propto}] \sin(v) = r_4\, cos(v)$$

By coefficient comparison

$$\underline{a}_{02x} + 2\,\underline{a}_{03x} + 3\,\underline{a}_{04x} = 0,$$

$$\propto \underline{a}_{11x}\, e^{\propto} + (1+\propto)\,\underline{a}_{13x}\, e^{\propto} - \propto \underline{a}_{12x}\, e^{-\propto} + (1 - \propto)\,\underline{a}_{14x}\, e^{-\propto} = r_4,$$

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$$\propto \underline{b}_{11x}\, e^{\propto} + (1+\propto)\, \underline{b}_{13x}\, e^{\propto} - \propto \underline{b}_{12x}\, e^{-\propto} + (1-\propto)\, \underline{b}_{14x}\, e^{-\propto} = 0.$$

With respect to y-axis $\quad \dfrac{\partial \underline{x}_y}{\partial u}(1,v) = r_4 \sin(v).$

We get

$$\underline{a}_{02y} + 2\,\underline{a}_{03y} + 3\,\underline{a}_{04y} = 0,$$

$$\propto \underline{a}_{11y}\, e^{\propto} + (1+\propto)\, \underline{a}_{13y}\, e^{\propto} - \propto \underline{a}_{12y}\, e^{-\propto} + (1-\propto)\, \underline{a}_{14y}\, e^{-\propto} = 0,$$

$$\propto \underline{b}_{11y}\, e^{\propto} + (1+\propto)\, \underline{b}_{13y}\, e^{\propto} - \propto \underline{b}_{12y}\, e^{-\propto} + (1-\propto)\, \underline{b}_{14y}\, e^{-\propto} = r_4.$$

At the end with respect to z-axis $\quad \dfrac{\partial \underline{x}_z}{\partial u}(1,v) = h_4.$

We found

$$\underline{a}_{02z} + 2\,\underline{a}_{03z} + 3\,\underline{a}_{04z} = h_4,$$

$$\propto \underline{a}_{11z}\, e^{\propto} + (1+\propto)\, \underline{a}_{13z}\, e^{\propto} - \propto \underline{a}_{12z}\, e^{-\propto} + (1-\propto)\, \underline{a}_{14z}\, e^{-\propto} = 0,$$

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$$\propto \underline{b}_{11z}\, e^{\propto} + (1+\propto)\, \underline{b}_{13z}\, e^{\propto} - \propto \underline{b}_{12z}\, e^{-\propto} + (1-\propto)\, \underline{b}_{14z}\, e^{-\propto} = 0.$$

Now by solving all of the above equations by taking $\propto = 1$ we obtained values of $\underline{a}_{01q}$, $\underline{a}_{02q}$, $\underline{a}_{03q}$; $\underline{a}_{04q}$, $\underline{a}_{11q}$, $\underline{a}_{13q}$, $\underline{a}_{12q}$, $\underline{a}_{14q}$, $\underline{b}_{11q}$, $\underline{b}_{13q}$, $\underline{b}_{12q}$, $\underline{b}_{14q}$. where $(q = x, y, z)$

For the geometric representation of the axisymmetric oblate spheroid as shown in Figure 3.2, only the upper hemisphere will be considered because this object is symmetric, the size r_i and height h_i the upper Hemisphere are determined by

$$r_i = a\cos(\emptyset) \quad \text{and} \quad h_i = b\sin(\emptyset). \tag{3.12}$$

where $\emptyset = [\,0, \pi\,]$, a and b are the radius of hemisphere on x-axis and y-axis respectively, generally the coordinate of each point on the boundary curves can be written as

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$$(x_0 + r_i \cos(v),\, y_0 + r_i \sin(v),\, z_0 + h_i). \tag{3.13}$$

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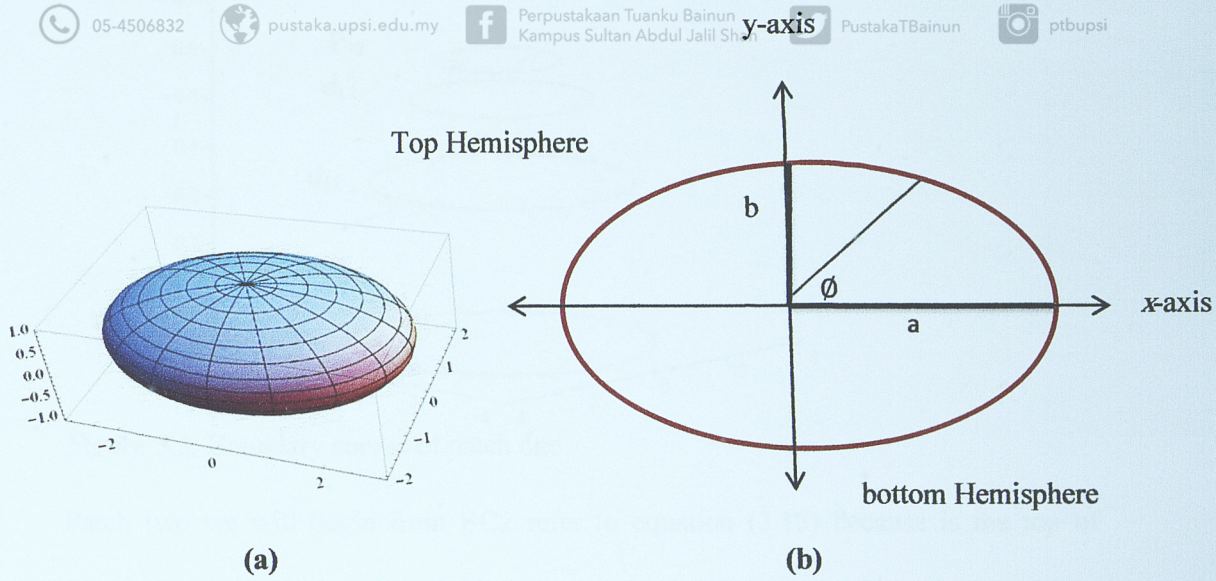


Figure 3.2. Hemisphere in 2D and 3D. (a) hemisphere in 3D, (b) hemisphere in 2D

where $0 \leq v \leq 2\pi$ and x_0 , y_0 , z_0 represent the center of hemisphere. Now let us consider the boundary curve used to generate first patch of the upper hemisphere with a = 3mm and b = 2mm. Generally BC1, BC2, BC3 and BC4 are given in equations (3.3), (3.7), (3.10) and (3.11) respectively in particular, the coordinate for boundary curves are

$$P_{11} = (x_0 + 3\cos(v),\ y_0 + 3\sin(v),\ z_0), \tag{3.14}$$

$$P_{12} = (x_0 + 1.2\cos(v),\ y_0 + 1.2\sin(v),\ z_0 + 0.58), \tag{3.15}$$

$$d_{11} = (x_0 + 0.1\cos(v),\ y_0 + 0.1\sin(v),\ z_0 + 0.01), \tag{3.16}$$

$$d_{12} = (x_0 + 0.2\cos(v),\ y_0 + 0.2\sin(v),\ z_0 + 1.4), \tag{3.17}$$

where P_{11} represent BC1 for patch one, P_{12} is BC2 for patch one, d_{11} is BC3 for patch one and d_{12} is BC4 for patch one, notice that BC1 is a base, BC2 is the top while BC3 and BC4 are the body for every shape, see Figure 3.3.

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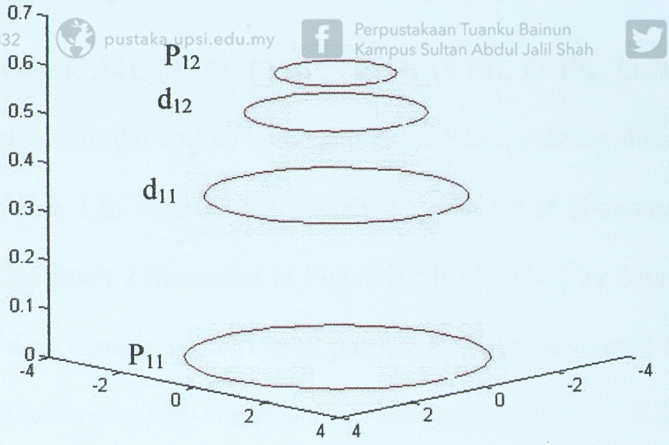


Figure 3.3. Boundary curves of patch one

Patch two we will begin from BC2 refer to equation (3.15) because is the top of Figure and come down to a point because we need to have a closed surface as shown in Figure 3.4. The BCs of patch two are given as

$$P_{21} = (x_0 + 1.2\cos(v),\ y_0 + 1.2\sin(v),\ z_0 + 0.58), \tag{3.18}$$

$$P_{22} = (x_0 + y_0 + z_0 + 0.36), \tag{3.19}$$

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$$d_{21} = (x_0 + 0.4\cos(v),\ y_0 + 0.4\sin(v),\ z_0 + (-0.8)), \tag{3.20}$$

$$d_{22} = (x_0 + 0.6\cos(v),\ y_0 + 0.6\sin(v),\ z_0 + 0.001). \tag{3.21}$$

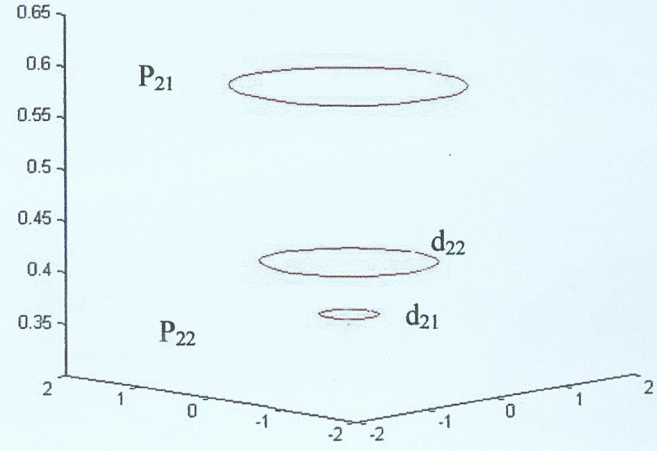


Figure 3.4. Boundary curves of patch two

By drawing equations (3.14), (3.15), (3.16), (3.17), (3.18), (3.19), (3.20) and (3.21), we get the top hemisphere of RBC as it shown in Figure 3.5.d. The condition

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of equations (3.14), (3.15), (3.16), (3.17), (3.18), (3.19), (3.20) and (3.21), can be utilized to obtain the bottom hemisphere. Each two patches share one boundary curve. Surface patch 1 in Figure 3.5.a, sketched by the set of boundary curves in Figure 3.3 and surface patch 2 illustrated in Figure 3.5.b, sketched by boundary curves in Figure 3.4, and then combine patch1 with patch 2 by boundary curve P_{12} or P_{21} as in Figure 3.5.c, we will get a new shape say top hemisphere Figure 3.5.d. By the same way of sketching top hemisphere just with negative sign of z-axis we will get the bottom of hemisphere see Figure 3.6.f.

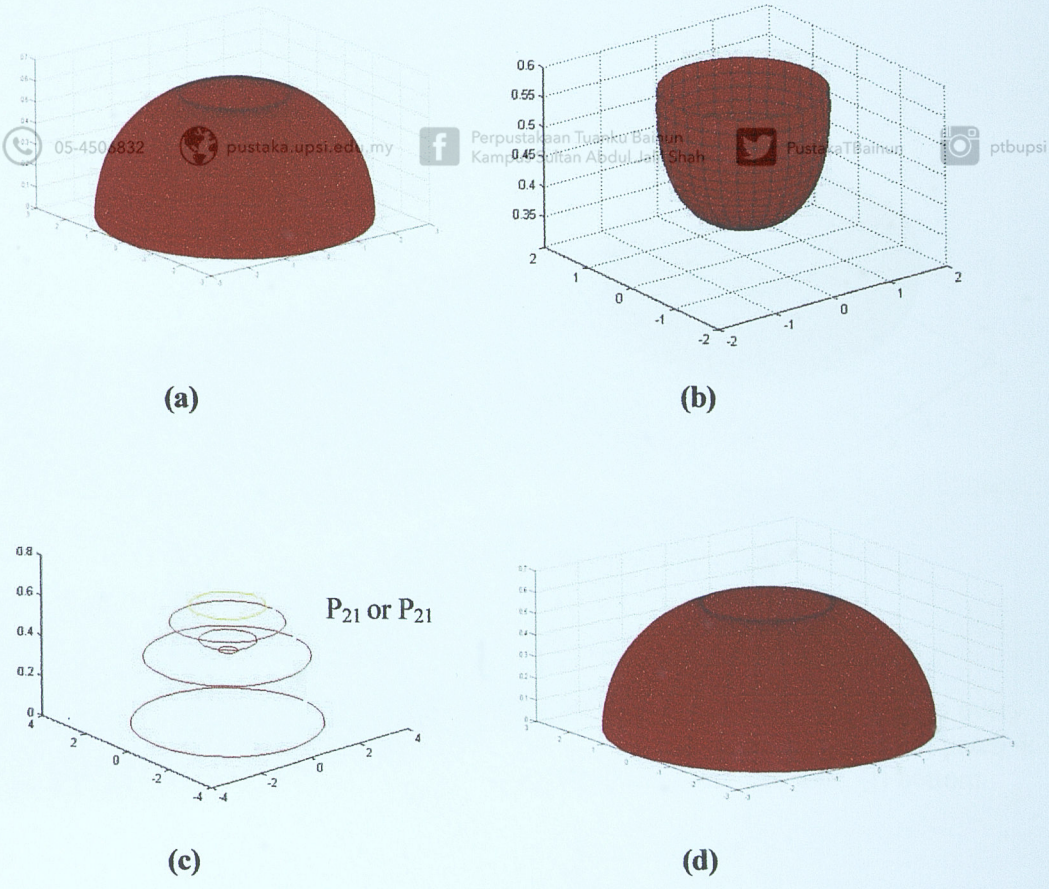
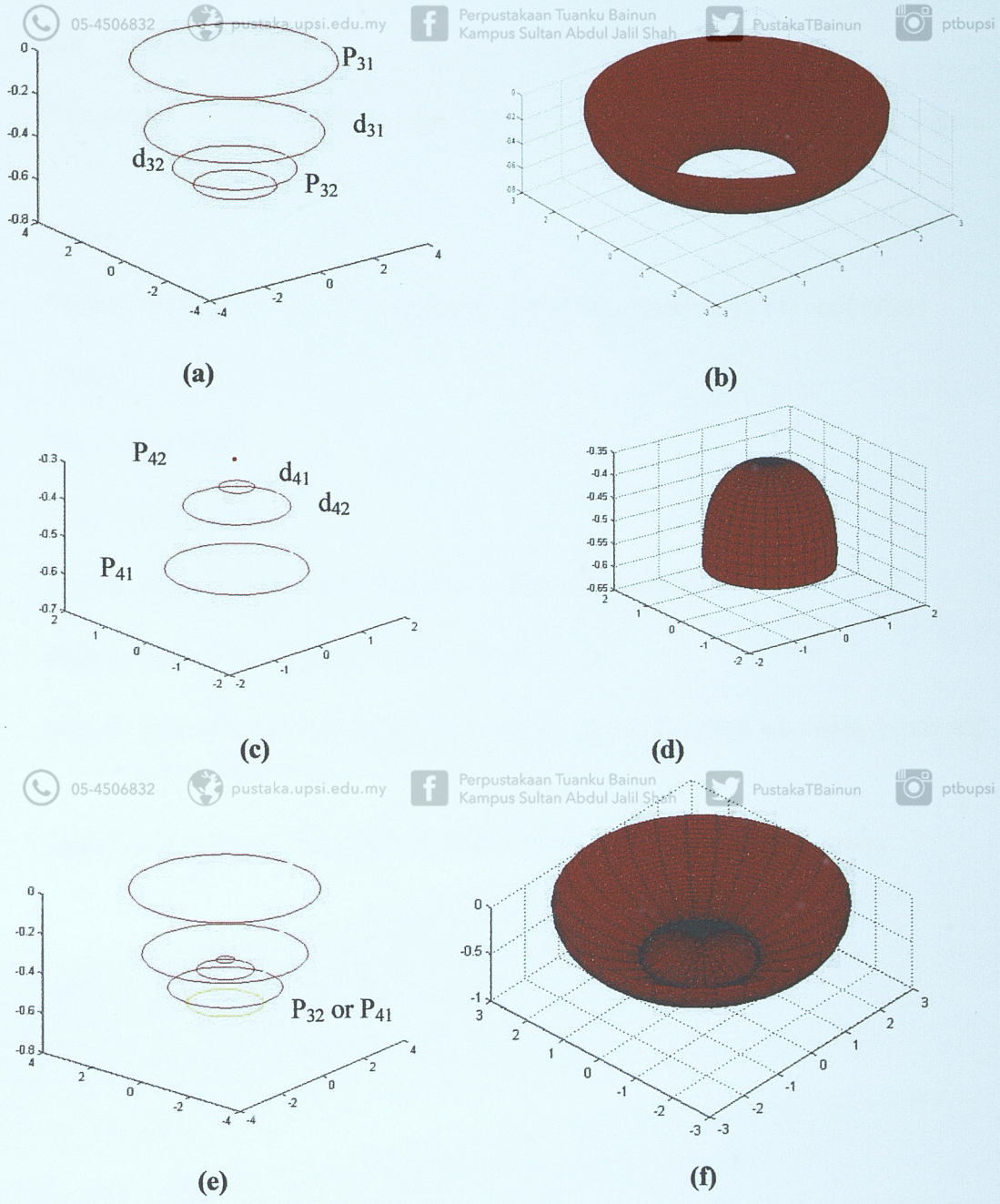


Figure 3.5. The top curves for a closed Hemisphere and corresponding surface shape to every patches. (a) patch 1, (b) patch 2, (c) combination between boundary curve one and two, (d) top hemisphere.

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Figure 3.6. The bottom curves for a closed hemisphere with surface shape to every patches. (a) boundary curves of patch 3, (b) patch 3, (c) boundary curves of patch 4, (d) patch 4, (e) combination between boundary curve three and four, (f) bottom Hemisphere.

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3.3 PSEUDOCODE

This pseudo code shows how to generate the shape of RBC, as can be seen in Figure 3.7.b.

Input:

Radius (r) and Height (h) from boundary conditions, equations (3.14) until (3.21)

Output

The shape of RBC

Step 1: set r and h.

Step 2: create matrices and vectors by coefficients in all BCs.

Step 3: set domain u from 0 to 1 and v from 0 to 2π.

Step 4: calculate matrices and vectors to get A_0, A_n and B_n which are z-axis, x-axis and y-axis respectively.

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Step 5: call surface to generate the shape of RBC on x-axis, y-axis and z-axis.

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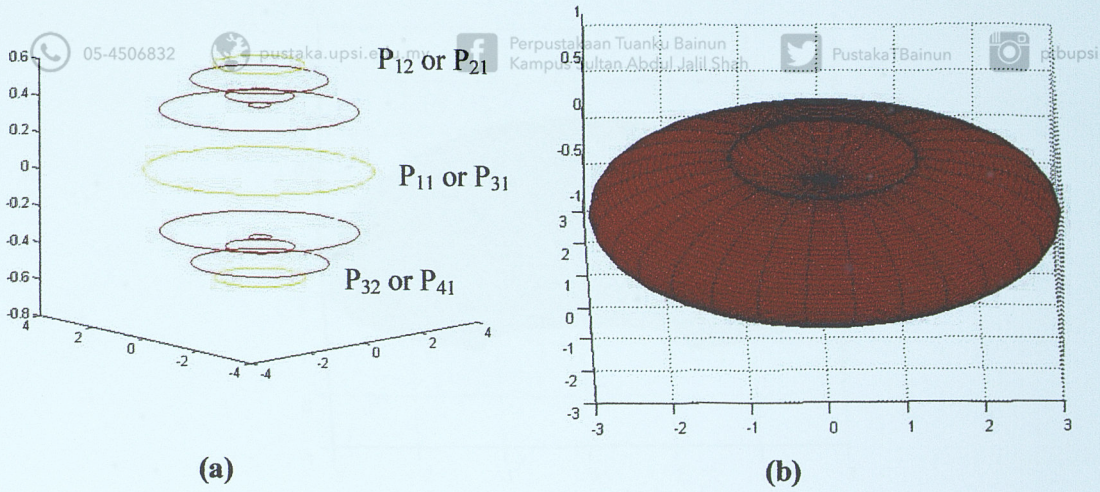


Figure 3.7. Red Blood Cell (MATLAB). (a) boundary curves, (b)resulting PDE surface of red blood cell.

Sketching whole RBC needs four PDEs and every PDE has four boundary conditions, each two PDE combined by a boundary curve therefore sketching RBC need 13 boundary curves, where 11 of them are circle but the other two are points. Depends on two variables u and v, where u begins from 0 to 1 while v from 0 to 2π and the Figure of RBC is symmetric about x-axis as it is shown in Figure 3.7.

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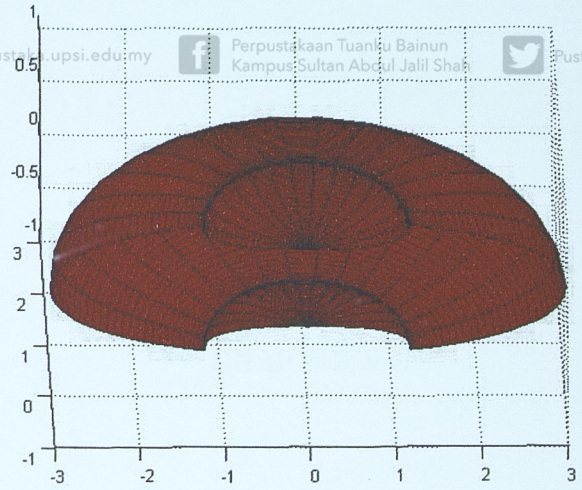


Figure 3.8. Half Red Blood Cell

Figure 3.8 is the half RBC and it is sketched by the same way of Figure 3.7.b, and also depends on two variables u and v, where u begin from 0 to 1 but v from 0 to π.

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3.4 CONCLUSION

This chapter focuses on the application of the analytic solution of a particular Elliptic PDE for designing a parametric representation of a RBC. Generally, four smooth surface patches generated by a fourth order PDE have been blended together to construct the shape using MATLAB, all boundary curves have been created after inputting it's boundary condition into program.

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CHAPTER 4

MODELLING OF SICKLE CELL

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4.1 INTRODUCTION

This chapter discusses the shapes of a sickle cell and explains how the shapes will be created by the PDE method. There are many shapes of sickle cells, so this research selected only two shapes.

Sickle cell Anemia is an illness of the blood caused by genetic unusual hemoglobin (the oxygen-carrying protein within the red blood cells). The abnormal

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hemoglobin causes distorted (sickled) red blood cells. The sickled red blood cells are breakable and prone to tear. When the number of the red blood cells decreases from rupture, sickle cell is the result. This condition is referred to as sickle cell (Kathleen and Julie, 2011).

4.2 SKETCHING THE SHAPES OF A SICKLE CELL ANEMIA

This section discusses how to use the PDE method for designing generic shapes of sickle cell. During this work 7 curves with two patches will be used to generate the first shape for sickle cell (shape 1), while 13 curves as well as four patches will used to generate another shape of sickle cell (shape 2), and all those curves are generated using Matlab. Every curve represents a circle of a given radius, height and center.

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As mentioned before, sickle cells have more than one shape and some of them are quite different from the others.

Two shapes of sickle cells will be sketched in this section. The first shape (Shape 1) will be sketched by using two PDEs, as shown in Figure 4.1. Each PDE has four boundary curves.

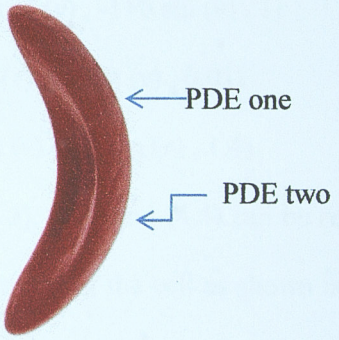


Figure 4.1. Sickle cell shape 1 (Saraf 2015)

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The other shapes will be sketched, by using four PDEs, as shown in Figure 4.2. In addition, each PDE has four boundary curves.

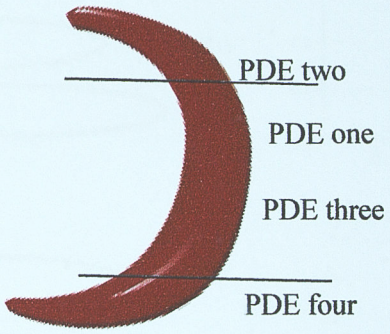


Figure 4.2. Sickle cell shape 2 (Robin 2015)

With respect to sketching sickle cell's shapes, equation (2.16) is solved by substituting equations (2.17), (2.18) and (2.19) into the equation (2.16) and solving it as in chapter three with different boundary conditions.

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In order to sketch the first shape of sickle cell (Sickle cell shape 1) Figure (4.1) four BCs are needed for each PDE. The boundary conditions of the first PDE (PDE one) are:

$$P_{11} = (0.5 \cos(v), 0.5 \sin(v), 0) \tag{4.1}$$

$$P_{12} = (-1+0 \cos(v), -1+0 \sin(v), 1) \tag{4.2}$$

$$d_{11} = (-1 \cos(v), -1 \sin(v), 0.01) \tag{4.3}$$

$$d_{12} = (0.1 \cos(v), 0.1 \sin(v), 1.4) \tag{4.4}$$

The conditions in equations (4.1), (4.2), (4.3) and (4.4) can be reflected to obtain the conditions corresponding to the lower part of the cell as shown in Figure (4.1), which is PDE two

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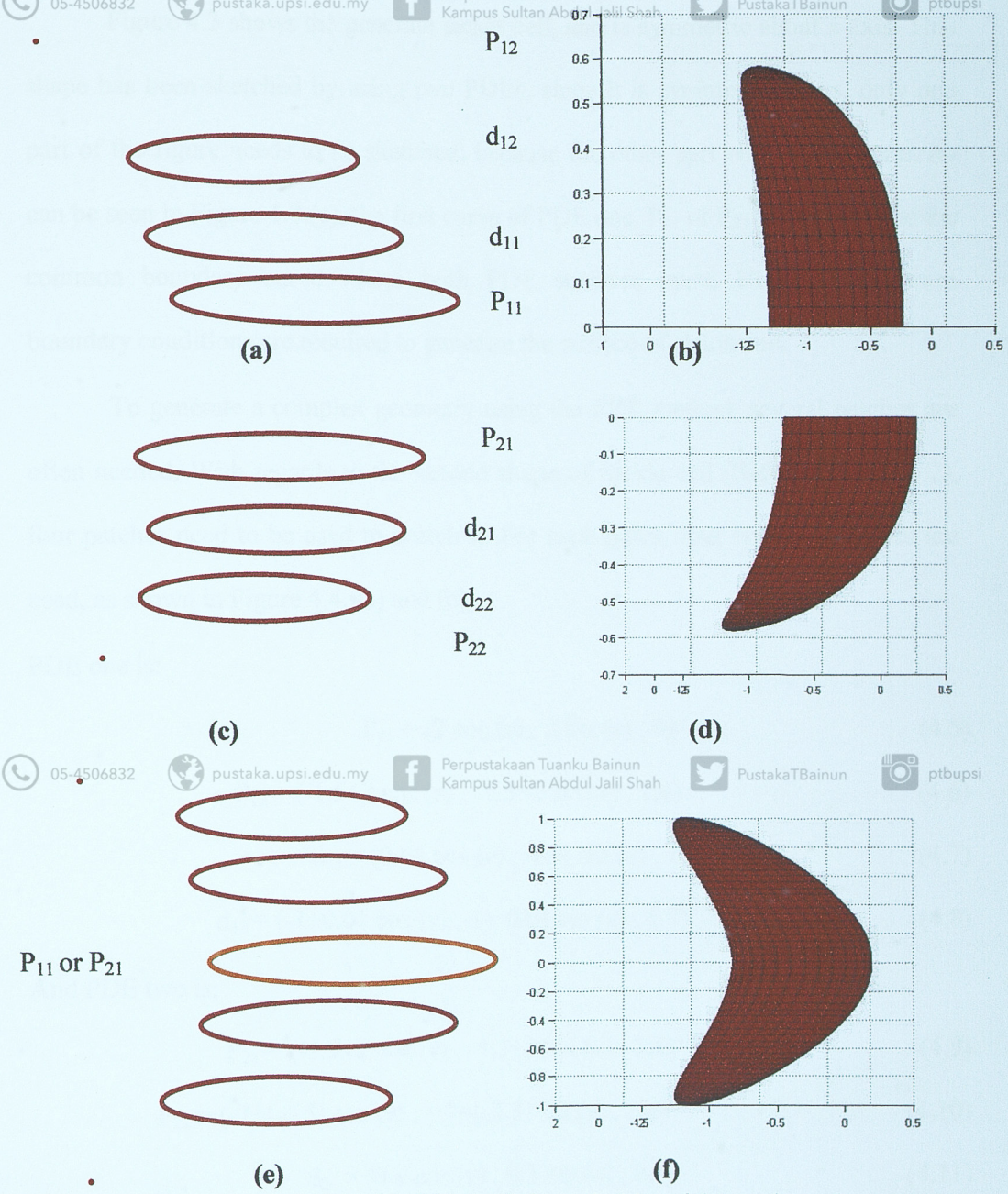


Figure 4.3. Geometric model of a sickle cell shape 1. (a) and (c) are boundary curves of patch one and patch two respectively. (b) and (d) are PDE surface of patch one and two respectively. (e) Boundary curves of a sickle cell shape 1. (f) Resulting surface of a sickle cell shape 1.

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Figure 4.3 shows the generate sickle cell, and is symmetric about x-axis. This shape has been sketched by using two PDEs, since it is symmetric. Thus, only one part of the figure needs to be sketched, because the other part will be the same. As can be seen in Figure 4.3.(e), the first curve of PDE one, P_{11} or P_{21} is chosen to be the common boundary curve where both PDE surfaces meet. Hence, only seven boundary conditions are required to generate the surface of sickle cell.

To generate a complex geometry using the PDE method, several patches are often needed. With regards to the second shape of Sickle cell (Sickle cell shape 2), four patches need to be used to sketch it. For each patch, four boundary curves are used, as shown in Figure 4.4. (a) and (c).

PDE one is:

$$P_{11} = (2\cos(v), 2\sin(v), 0) \tag{4.5}$$

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$$P_{12} = (-1.5+2\cos(v), -1.5+2\sin(v), 1.4) \tag{4.6}$$

$$d_{11} = (0.04\cos(v), 0.04\sin(v), 1) \tag{4.7}$$

$$d_{12} = (-1+0.01\cos(v), -1+0.01\sin(v), 1.7) \tag{4.8}$$

And PDE two is:

$$P_{21} = (-1.5+2\cos(v), -1.5+2\sin(v), 1.4) \tag{4.9}$$

$$P_{22} = (-7+(-0.5)\cos(v), (-7+(-0.5)\sin(v), 2.8) \tag{4.10}$$

$$d_{21} = (0.3\cos(v), 0.3\sin(v), 0.1) \tag{4.11}$$

$$d_{22} = (-5+(-6)\cos(v), -5+(-6)\sin(v), -0.1) \tag{4.12}$$

The conditions in Equations (4.5), (4.6), (4.7), (4.8), (4.9), (4.10), (4.11) and (4.12) can be reflected to obtain the conditions corresponding to the lower part of the cell as shown in Figure 4.5.f, which is PDE three and four.

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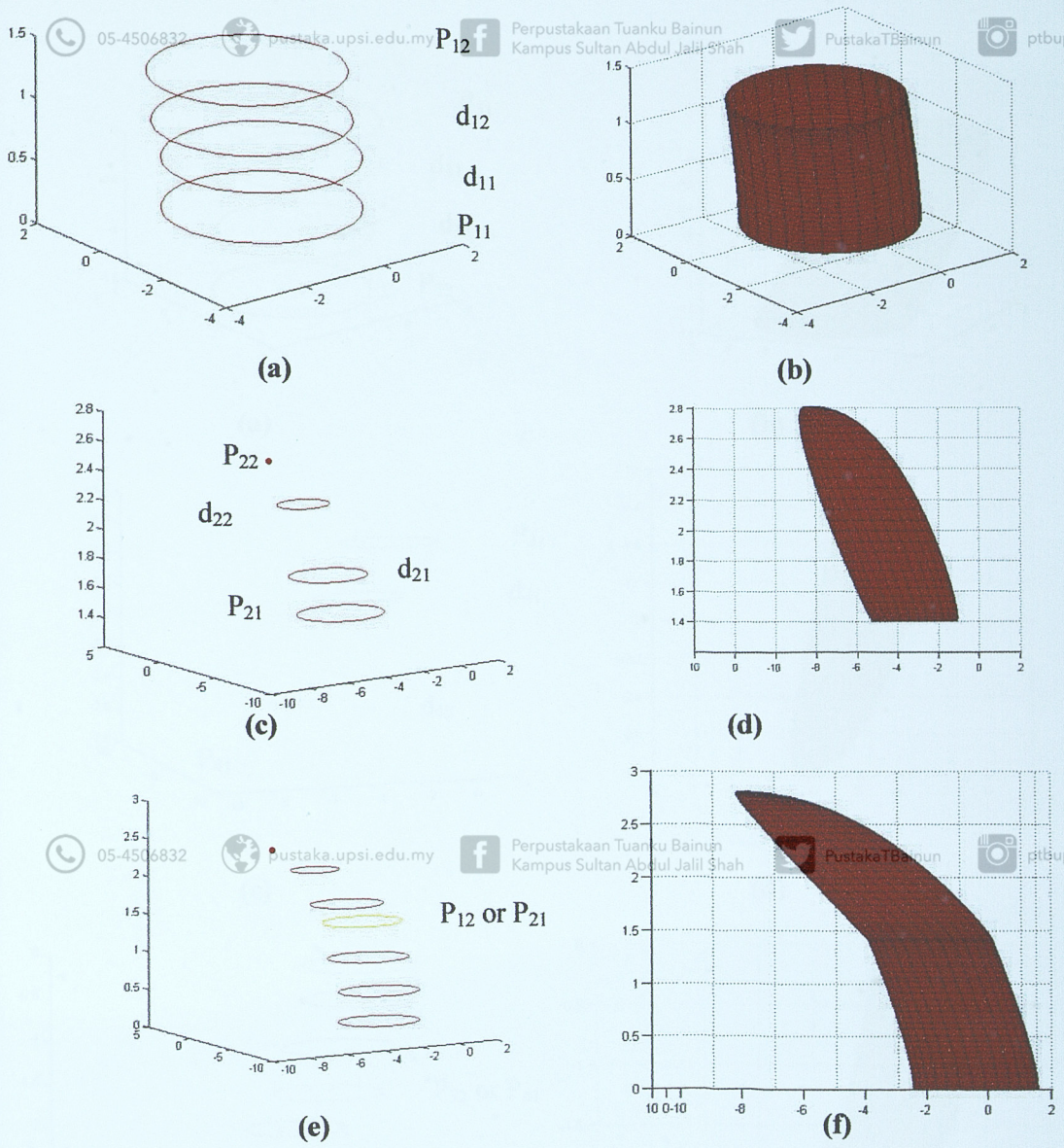


Figure 4.4. The upper part of Sickle cell shape 2. (a), (c) and (e) are boundary curves of patch one, patch two and both of them respectively,(b) and (d) are PDE surface of one and two respectively, (f) Combination between patch one and two.

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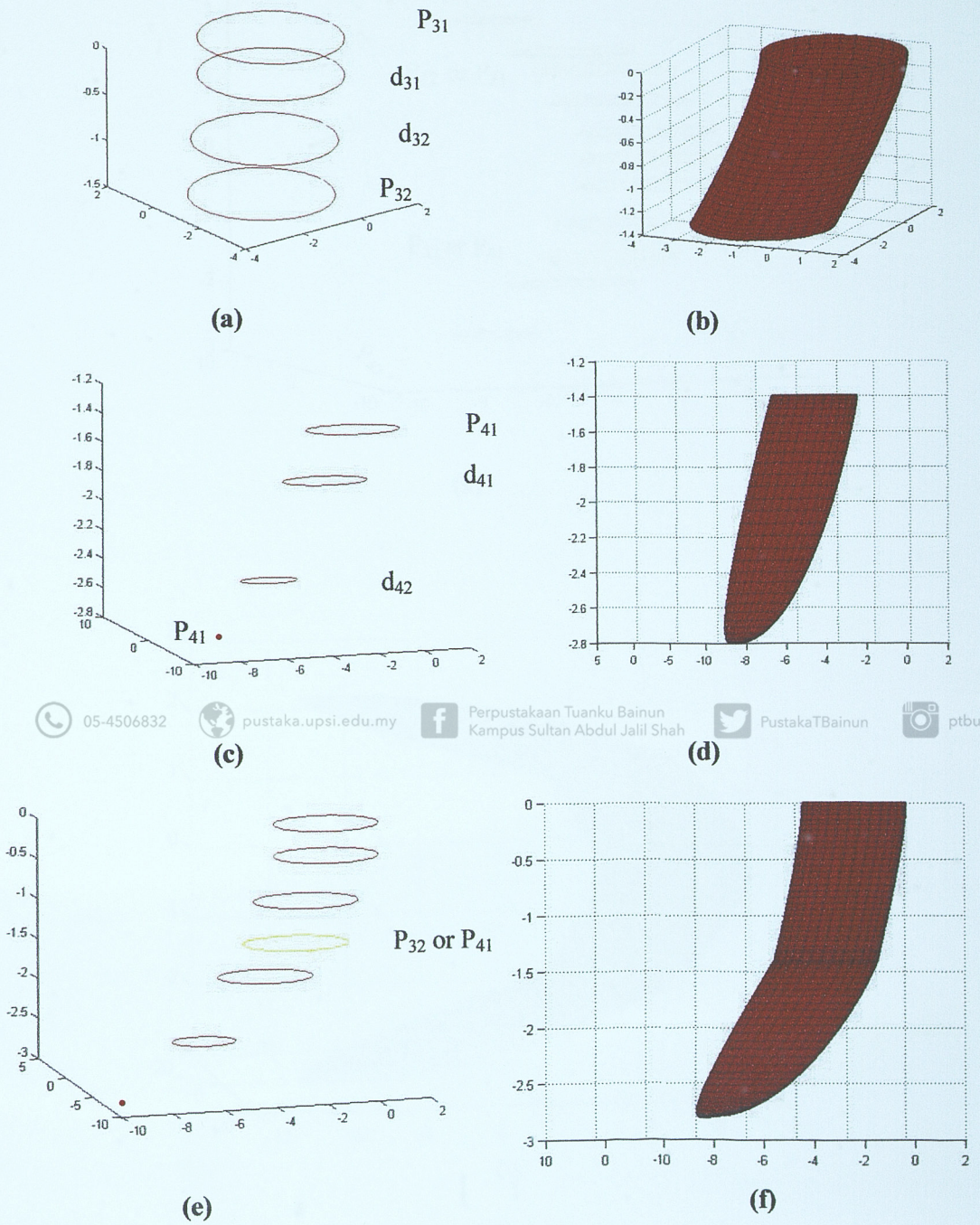
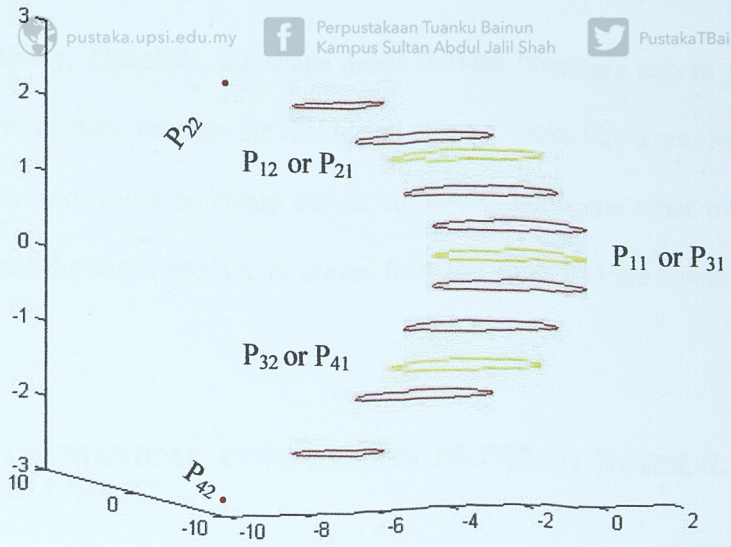


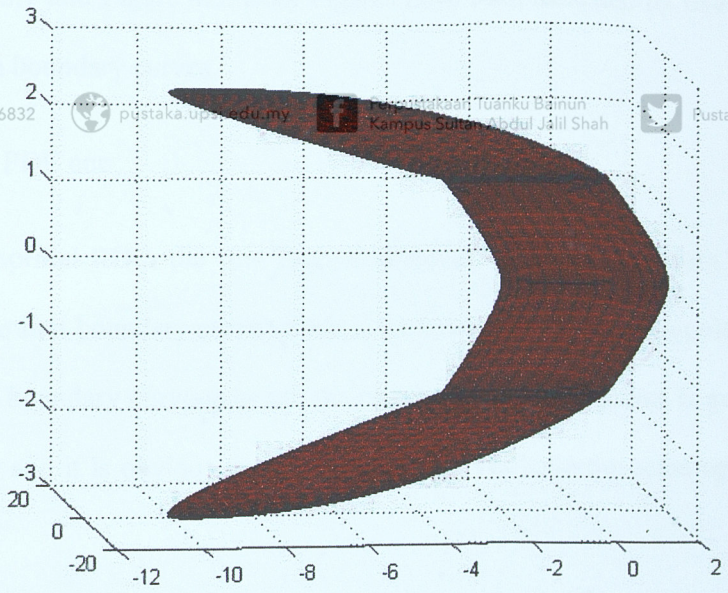
Figure 4.5. The lower part of Sickle cell shape 2. (a), (c) and (e) are boundary curves of patch three, patch four and both of them respectively, (b) and (d) PDE surface of patch three and four respectively, (f) Combination between patch three and four.

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(a)



(b)

Figure 4.6. The PDE - based sickle cell shape 2. (a) boundary curves (b) the resulting PDE surface of a sickle cell.

Figure 4.6 shows the resulting shape of a sickle cell. As one can see in Figure 4.6.a, boundary curves P_{11}, P_{14} and P_{32} are used to be common boundary curves between patch one and patch three, patch one and patch two, patch three and patch

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four respectively. Therefore, the shape needs thirteen boundary curves for sketching with different centers, because the Sickle cell shape 2 looks like a crescent shape or a Banana. Eleven of those boundary curves are circles while the other two curves are points because the shape needs to be closed from the upper and the lower.

4.3 MATHEMATICAL COMPARISON BETWEEN NORMAL RBC AND SICKLE CELL'S RBC

Since sickle cell has more than one shape, therefore, one of them has been selected to be compared with normal RBC Mathematically. Thus, the comparison is between Figure 3.7 and Figure 4.6. Both Figures have been sketched by using four PDEs and thirteen boundary curves.

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- PDE one

In normal RBC, the first PDE is sketched with four boundary curves. It begins with the first boundary curve which is the base and it has the biggest radius. From the bottom boundary curve goes up, the radius of boundary curves is getting smaller and smaller and it is obvious that all boundary curves have the same center, as in Figure 3.5.a.

In Sickle cell's RBC, the first PDE is sketched with four boundary curves. It begins with the first boundary curve, which has smaller radius than the radius of first boundary curve in normal RBC. Then it goes up. Those boundary curves have different centers and they move toward negative x-axis (see Figure 4.4.b).

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- PDE two

With respect to normal RBC, the second PDE begins from the top boundary curve of the first PDE and moves downward. When the second PDE comes down, the radius of boundary curves will be smaller and smaller until it becomes only a point, which is the fourth boundary curve, because it needs to be a closed surface. All boundary curves in the second PDE have the same center, as in the first PDE (see Figure 3.5.b).

In Sickle cell's RBC, the second PDE begins from the top boundary curve of the first PDE and it continues going up. The last boundary curve is a point and all boundary curves have different center to negative x-axis (see Figure 4.4.d).

Table 4.1

Comparison between Normal and Sickle cell's RBC in the first and the second PDE

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Normal			**Sickle**	
boundary curves	**PDE one**			
	r	**H**	**R**	**h**
One	**3**	**0**	**2**	**0**
Two	**1.2**	**0.58**	**2**	**1.4**
Three	**2.6**	**0.33**	**2**	**0.4**
Four	**1.8**	**0.5**	**2**	**0.9**
	PDE two			
One	**1.2**	**0.58**	**2**	**1.4**
Two	**0**	**0.35**	**0**	**2.8**
Three	**0.9**	**0.41**	**1.8**	**1.72**
Four	**0.3**	**0.36**	**1.2**	**2.33**

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Table 4.1 shows the comparison between radius (r) and height (h) of each boundary curve for the first and the second PDEs of a normal and sickle red cell.

As it is mentioned before, the normal RBC and sickle red cell are symmetric. In terms of the x-axis, the first and the second PDEs have been sketched above the x-axis (positive y) while the third and the fourth PDEs are sketched below the x-axis (negative y). It will be noticed that the third and the fourth PDEs have the same properties as of the first and the second PDEs, respectively, in normal RBC and also in sickle red cell (see Figure 3.7 and Figure 4.6).

In order to express the mathematical relationship between the radius and the height of a normal RBC and a sickle cell, the point of radius and height will be sketched on x-axis and y-axis respectively and then plotting those points.

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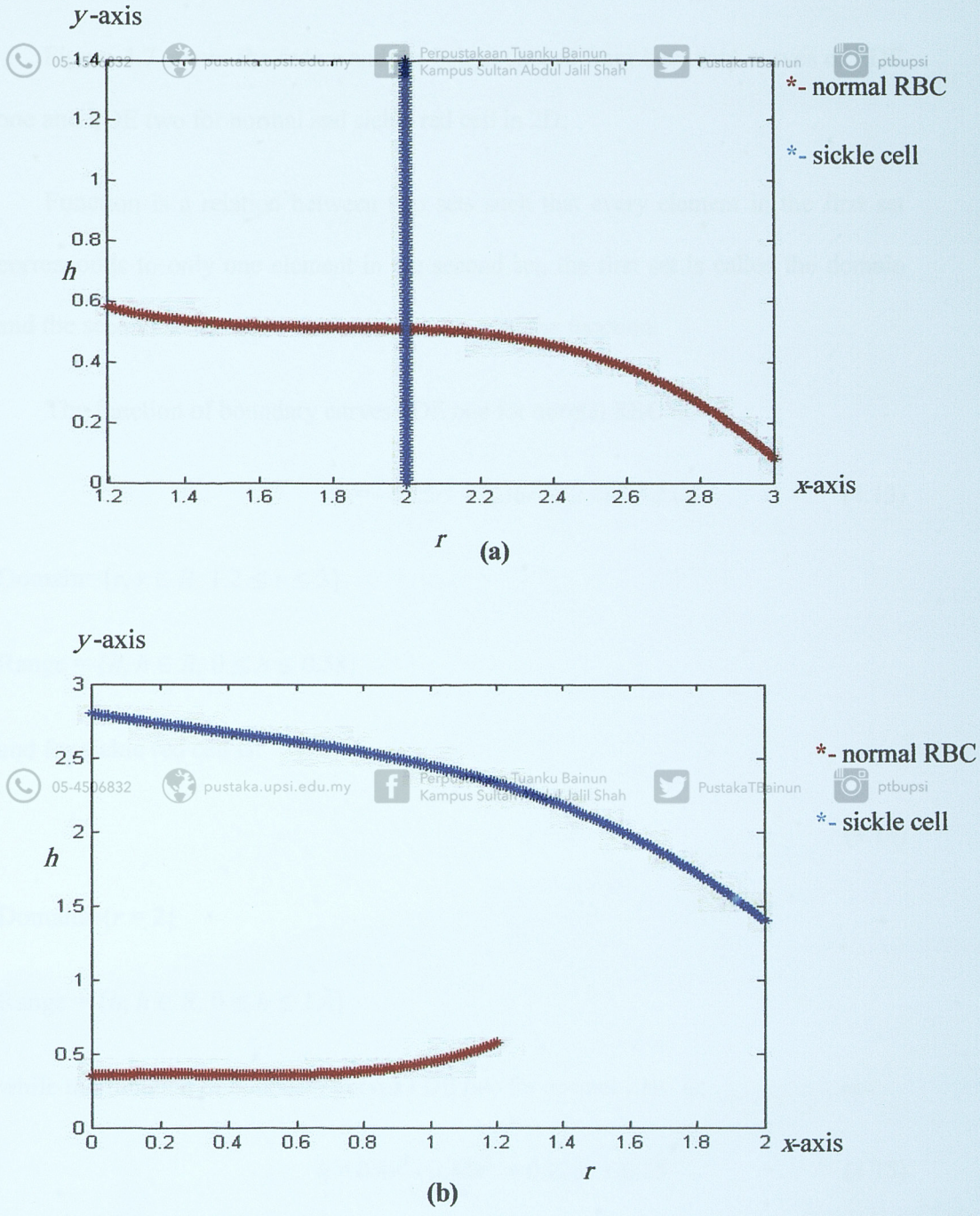


Figure 4.7. plotting the dimension of each BCs of first and the second PDE for normal and sickle red cell in 2D. (a) radius and height for PDE one for normal RBC and for sickle red cell (b) radius and height for PDE two for normal RBC and for sickle red cell.

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Figure 4.7 shows the radius and the height chosen from boundary curves of PDE one and PDE two for normal and sickle red cell in 2D.

Function is a relation between two sets such that every element in the first set corresponds to only one element in the second set, the first set is called the domain and the second set is called co-domain or range of the function.

The function of boundary curves PDE one for normal RBC is:

$$h = -0.25r^3 + 1.36r^2 - 2.48r + 2.03 \tag{4.13}$$

Domain=$\{r, r \in R,\ 1.2 \leq r \leq 3\}$

Range = $\{h, h \in R,\ 0 \leq h \leq 0.58\}$

and for sickle red cell is:

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$$r = 2 \tag{4.14}$$

Domain=$\{r = 2\}$

Range = $\{h, h \in R,\ 0 \leq h \leq 1.4\}$

while the function of boundary curves PDE two for normal RBC is:

$$h = 0.4r^3 - 0.43r^2 + 0.125r + 0.35 \tag{4.15}$$

Domain=$\{r, r \in R,\ 0 \leq r \leq 1.2\}$

Range = $\{h, h \in R,\ 0.58 \leq h \leq 0.35\}$

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$$h = -0.191r^3 + 0.2257r^2 - 0.3875\,r + 2.8 \qquad (4.16)$$

Domain=$\{r, r \in R,\ 0 \leq r \leq 2\}$

Range = $\{h, h \in R,\ 1.4 \leq h \leq 2.8\}$

The domain of a function is the complete set of possible values of the independent variable. In general, we determine the domain of each function by looking for those values of the independent variable which we are allowed to use.

The range of a function is the complete set of all possible resulting values of the dependent variable, after we have substituted the domain.

It is shown that the value of height, h and radius, r of normal RBC and sickle cells can be obtained from these equations. Therefore, Equations (4.13),(4.15) and (4.14), (4.16) are capable of predicting the dimensions of both cells respectively. This is useful to sketch the 2D and 3D shapes of normal RBC and sickle cells in future.

4.4 CONCLUSION

Two shapes of sickle RBC have been sketched by PDE method using MATLAB program, and one of them was mathematically compared with normal RBC. This chapter also shows graph of radius and height from every boundary curves in PDE one and two for normal and sickle cell. If any value of r in particular domain is selected and substitute into that particular equation, the value of h can be obtained and it will be in the range of that equation.

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CHAPTER 5

CONCLUSION AND FUTURE WORK

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5.1 CONCLUSION

During this work, the equation of the PDE method, in general, was solved by using four boundary conditions. In addition to that, by using the same method (PDE method), red blood cell (RBC) was sketched by utilizing four partial differential equations (PDEs) such that each PDE had four boundary conditions (BCs). In such a way that the third and the first PDEs were the same, furthermore, the fourth and the second PDEs were similar. This was because of the graph of RBC, which was symmetric about x-axis and by applying this method, RBC was cut into two pieces

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and from this another shape was generated and this shape was obtained by the help of MATLAB software.

There are many shapes of anemia disease. In this study, two shapes of sickle red cells were selected to be sketched by using the PDE method. One of them was sketched by using two PDEs, whereas, the other was sketched by using four PDEs. Sketching and finding the equations of normal red blood cells and sickle red cell have been discovered by using the radius and the height of each boundary curves of PDE one and PDE two.

5.2 DISCUSSION

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The differential geometry of a normal red blood cell is treated using Cassinian oval for modeling its shape. In this connection an explicit parameterization is found by Jacobian elliptic function of the usual polar coordinates as shown in figure 2.1.

Modeling, the shape of red blood cell and two shapes of sickle red cells geometrically, by using an elliptic partial differential equation (PDE) within four boundary conditions.

This thesis has achieved two main objectives. Firstly, by using the PDE method, the normal RBC has been modeled. Secondary, sketching the disease that changes the shape of normal RBC is called sickle red cell Anemia. The shape of normal RBC has been modeled by the combination of four patches as well as designing two shapes of sickle red cell Anemia. One of them is sketched by using two linked patches while the other one has been sketched by utilizing four combined patches. Also, the shapes of sickle red cells were generated by changing radius, height

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and center of the normal RBC and one of them has also been compared mathematically with normal RBC as shown in table 4.1.

PDE method was used in this work in order to model normal RBC and sickle cell, while in literature review Cassinian oval was used for modeling normal RBC since it takes a lot of time, it uses many equations and it is concerned with the determination of explicit parametric equations of several plane curves.

What it used in this work is better and easier than what it used in literature review because PDE method depends on the boundary conditions, boundary curves have been achieved from boundary conditions and from this the required shapes have been attained.

The consequences of this work are obtaining and finding the equations of normal RBC and sickle red cell as well as their domain and range. These radius and height have been achieved from boundary curves and it is known that these boundary curves created by a boundary condition.

5.3 FUTURE WORK

This study motivates a number of currently exposed enquiries for future exploration projects and directions. Future orientations of this work will be exploring of the following enquires which will be finding the area and volume of the normal red blood cells. Moreover, we will attempt to search for all the diseases that affect the shape of RBCs.

In addition, based on the work carried out, in future researcher will improve his work to get a better shapes which will be more and more similar to real shapes of

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healthy red blood cells and of whom who has the disease that affects the shape of RBCs can be developed.

PDE with boundary conditions has been solved via separation of variable. The researcher will attempt to do the same process via some other methods such as integral transform, finite element method and any other possible methods for doing this process in order to model the normal RBC and sickle red cells and so as to sketching their shapes, in this thesis, MATLAB program has been used. In future, some other mathematical programs will be used for sketching normal RBC and sickle red cell such as Maple and MathWorks.

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APPENDIX

Appendix A: Programming

This programme for sketching every figure that can be solved by PDE-method depending on matrix because the programme used *surf* and the variables that are used in *surf* must be as matrix neither scalar nor vector and since the programme could used more than one figure (one PDE) so it needs to input *hold on* for combining all figures, so the out put of this program is the shape of RBC Figure (3.7.b).

```
% the variables of boundary conditions
% PDE one
rx1=3;                                  % radius of cirlce one x-axis
ry1=3;                                  % radius of cirlce one y-axis
h1=0;                                   % high of cirlce one or z-axis
rx2=0.1;
ry2=0.1;
h2=0.01;
rx3=0.2;
ry3=0.2;
h3=1.4;
rx4=1.2;
ry4=1.2;
h4=0.58;
xi=1;                                   % xi and yi are ∝
yi=1;
B=[1 0 0 0;1 1 1 1;0 1 0 0;0 1 2 3];
%coefficient of a01, a02, a03 and a04 in all BC represent Xz
B1=[0;0;0;0];                           % center of x in all BC
B2=[0;0;0;0];                           % center of y in all BC
B3=[h1;h4;h3;h2];                       % center of z in all BC
j1=B\B1;
j2=B\B2;
j3=B\B3;
J1=[1 0 1 0;exp(xi) exp(xi) exp(-xi) exp(-xi);xi 1 -xi 1;
    xi*exp(xi) (1+xi)*exp(xi) -xi*exp(-xi) (1-xi)*exp(-
xi)];                   %coefficient of a11, a12, a13 and a14 in all BC represent Xx
L1=[rx1;rx4;rx3;rx2];
d=J1\L1;
J2=[1 0 1 0;exp(yi) exp(yi) exp(-yi) exp(-yi);yi 1 -yi 1;
    yi*exp(yi) (1+yi)*exp(yi) -yi*exp(-yi) (1-yi)*exp(-
yi)];                   %coefficient of a11, a12, a13 and a14 in all BC represent Xy
L2=[ry1;ry4;ry3;ry2];
c=J2\L2;
e=0:1/30:1;
u=e';
v=0:2*pi/30:2*pi;
X=j1(1)+(j1(2)*u)+(j1(3)*u.^2)+(j1(4)*u.^3);          %A0
Y=j2(1)+(j2(2)*u)+(j2(3)*u.^2)+(j2(4)*u.^3);
Z=j3(1)+(j3(2)*u)+(j3(3)*u.^2)+(j3(4)*u.^3);
```

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```
xn=(d(1).*exp(xi*u))+(d(2).*u.*exp(xi*u))+(d(3).*exp(-
xi*u))+(d(4).*u.*exp(-xi.*u));                          %An
yn=(c(1).*exp(yi*u))+(c(2).*u.*exp(yi*u))+(c(3).*exp(-
yi*u))+(c(4).*u.*exp(-yi.*u));                          %Bn
x=X*ones(size(v))+xn*cos(v);
y=Y*ones(size(v))+yn*sin(v);
z=Z*ones(size(v));
surf(x,y,z)
colormap([1   0 0; 1   0   0])                    % red color
hold on                                % combine more than one figure
% PDE two
rx4=1.2;
ry4=1.2;
h4=0.58;
rx5=0;
ry5=0;
h5=0.36;
rx6=0.4;
ry6=0.4;
h6=-0.8;
rx7=0.6;
ry7=0.6;
h7=0.001;
xi=1;
yi=1;
B=[1 0 0 0;1 1 1 1;0 1 0 0;0 1 2 3];
B1=[0;0;0;0];
B2=[0;0;0;0];
B3=[h4;h5;h6;h7];
j1=B\B1;
j2=B\B2;
j3=B\B3;
J1=[1 0 1 0;exp(xi) exp(xi) exp(-xi) exp(-xi);xi 1 -xi 1;
    xi*exp(xi) (1+xi)*exp(xi) -xi*exp(-xi) (1-xi)*exp(-
xi)];
L1=[rx4;rx5;rx6;rx7];
d=J1\L1;
J2=[1 0 1 0;exp(yi) exp(yi) exp(-yi) exp(-yi);yi 1 -yi 1;
    yi*exp(yi) (1+yi)*exp(yi) -yi*exp(-yi) (1-yi)*exp(-
yi)];
L2=[ry4;ry5;ry6;ry7];
c=J2\L2;
e=0:1/30:1;
u=e';
v=0:2*pi/30:2*pi;
X=j1(1)+(j1(2)*u)+(j1(3)*u.^2)+(j1(4)*u.^3);
Y=j2(1)+(j2(2)*u)+(j2(3)*u.^2)+(j2(4)*u.^3);
Z=j3(1)+(j3(2)*u)+(j3(3)*u.^2)+(j3(4)*u.^3);
```

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```
xn=(d(1).*exp(xi*u))+(d(2).*u.*exp(xi*u))+(d(3).*exp(-
xi*u))+(d(4).*u.*exp(-xi.*u));
yn=(c(1).*exp(yi*u))+(c(2).*u.*exp(yi*u))+(c(3).*exp(-
yi*u))+(c(4).*u.*exp(-yi.*u));
x=X*ones(size(v))+xn*cos(v);
y=Y*ones(size(v))+yn*sin(v);
z=Z*ones(size(v));
surf(x,y,z)
% PDE three
rx1=3;
ry1=3;
h1=0;
rx2=0.1;
ry2=0.1;
h2=0.01;
rx3=0.2;
ry3=0.2;
h3=1.4;
rx4=1.2;
ry4=1.2;
h4=0.58;
xi=1;
yi=1;
B=[1 0 0 0;1 1 1 1;0 1 0 0;0 1 2 3];
B1=[0;0;0;0];
B2=[0;0;0;0];
B3=[h1;h4;h3;h2];
j1=B\B1;
j2=B\B2;
j3=B\B3;
J1=[1 0 1 0;exp(xi) exp(xi) exp(-xi) exp(-xi);xi 1 -xi 1;
    xi*exp(xi) (1+xi)*exp(xi) -xi*exp(-xi) (1-xi)*exp(-
xi)];
L1=[rx1;rx4;rx3;rx2];
d=J1\L1;
J2=[1 0 1 0;exp(yi) exp(yi) exp(-yi) exp(-yi);yi 1 -yi 1;
    yi*exp(yi) (1+yi)*exp(yi) -yi*exp(-yi) (1-yi)*exp(-
yi)];
L2=[ry1;ry4;ry3;ry2];
c=J2\L2;
e=0:1/30:1;
u=e';
v=0:2*pi/30:2*pi;
X=j1(1)+(j1(2)*u)+(j1(3)*u.^2)+(j1(4)*u.^3);
Y=j2(1)+(j2(2)*u)+(j2(3)*u.^2)+(j2(4)*u.^3);
Z=j3(1)+(j3(2)*u)+(j3(3)*u.^2)+(j3(4)*u.^3);
xn=(d(1).*exp(xi*u))+(d(2).*u.*exp(xi*u))+(d(3).*exp(-
xi*u))+(d(4).*u.*exp(-xi.*u));
yn=(c(1).*exp(yi*u))+(c(2).*u.*exp(yi*u))+(c(3).*exp(-
yi*u))+(c(4).*u.*exp(-yi.*u));
```

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```
x=X*ones(size(v))+xn*cos(v);
y=Y*ones(size(v))+yn*sin(v);
z=-Z*ones(size(v));
surf(x,y,z)
colormap([1   0 0; 1  0  0])
hold on
% PDE four
rx4=1.2;
ry4=1.2;
h4=0.58;
rx5=0;
ry5=0;
h5=0.36;
rx6=0.4;
ry6=0.4;
h6=-0.8;
rx7=0.6;
ry7=0.6;
h7=0.001;
xi=1;
yi=1;
B=[1 0 0 0;1 1 1 1;0 1 0 0;0 1 2 3];
B1=[0;0;0;0];
B2=[0;0;0;0];
B3=[h4;h5;h6;h7];
j1=B\B1;
j2=B\B2;
j3=B\B3;
J1=[1 0 1 0;exp(xi) exp(xi) exp(-xi) exp(-xi);xi 1 -xi 1;
    xi*exp(xi) (1+xi)*exp(xi) -xi*exp(-xi) (1-xi)*exp(-
xi)];
L1=[rx4;rx5;rx6;rx7];
d=J1\L1;
J2=[1 0 1 0;exp(yi) exp(yi) exp(-yi) exp(-yi);yi 1 -yi 1;
    yi*exp(yi) (1+yi)*exp(yi) -yi*exp(-yi) (1-yi)*exp(-
yi)];
L2=[ry4;ry5;ry6;ry7];
c=J2\L2;
e=0:1/30:1;
u=e';
v=0:2*pi/30:2*pi;
X=j1(1)+(j1(2)*u)+(j1(3)*u.^2)+(j1(4)*u.^3);
Y=j2(1)+(j2(2)*u)+(j2(3)*u.^2)+(j2(4)*u.^3);
Z=j3(1)+(j3(2)*u)+(j3(3)*u.^2)+(j3(4)*u.^3);
xn=(d(1).*exp(xi*u))+(d(2).*u.*exp(xi*u))+(d(3).*exp(-
xi*u))+(d(4).*u.*exp(-xi.*u));
yn=(c(1).*exp(yi*u))+(c(2).*u.*exp(yi*u))+(c(3).*exp(-
yi*u))+(c(4).*u.*exp(-yi.*u));
x=X*ones(size(v))+xn*cos(v);
y=Y*ones(size(v))+yn*sin(v);
```

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```
z=-Z*ones(size(v));
surf(x,y,z)
```

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